

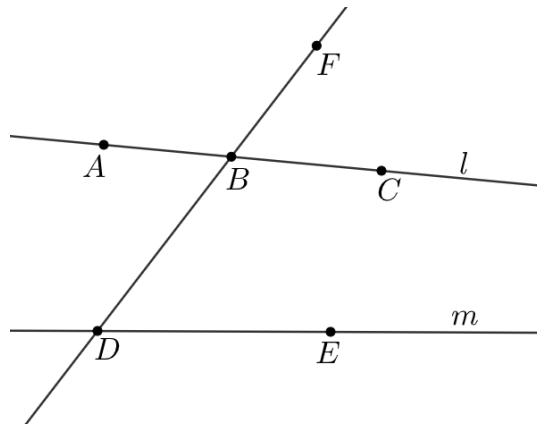
Lesson 1.15

Triangle Theorems

Geometry GT

Recall

In the figure below, lines l and m are *not* parallel and have been cut by a transversal.



Dylan believes $\angle CBF$ is congruent to $\angle BDE$ because they are corresponding angles and a translation along the directed line segment from B to C would take one angle onto the other. Here are his reasons:

- The translation takes B onto D , so the image of B is D
- The translation takes E somewhere on ray \overrightarrow{DB} because it would need to be translated by a distance greater than BD to land on the other side of D
- The image of C has to land somewhere on line m because translations take lines to parallel lines and line m is the only line parallel to l that goes through B'
- The image of C , call it C' , has to land on the right side of \overleftrightarrow{BD} or else $\overleftrightarrow{CC'}$ wouldn't be parallel to the directed line segment from B to D

Are the statements true or false? For any false statement, explain why it is not true.

Explore

Use a straightedge to create $\triangle ABC$ and label the three angle measures as a° , b° , and c° . Use either a straightedge and compass or paper folding to mark the midpoints of two of the sides, and extend the third side in both directions to make a line.

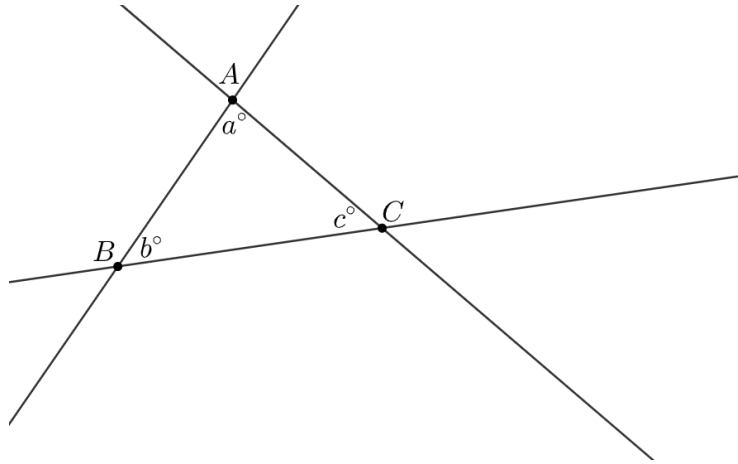
Use what you know about rotations to create a line parallel to the line you made that goes through the opposite vertex. What is the value of $a + b + c$? Explain your reasoning.

Theorem

Triangle Angle Sum Theorem: the three angle measures of any triangle always sum to 180°

Discuss

Here is $\triangle ABC$ with angle measures a° , b° , and c° . Each side has been extended to a line.



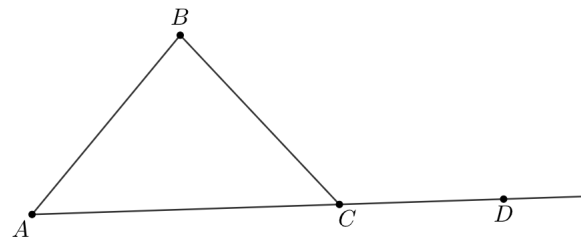
Translate $\triangle ABC$ along the directed line segment from B to C to make $\triangle A'B'C'$ and label the measures of the angles. Translate $\triangle A'B'C'$ along the directed line segment from A' to C' to make $\triangle A''B''C''$ and label the measures of the angles.

Label the measures of the angles that meet at C . Explain your reasoning. What is the value of $a + b + c$?

Demonstrate

Here is $\triangle ABC$ with point D on \overrightarrow{AC} (but not between A and C).

Explain how you know $m\angle BAC + m\angle ABC = m\angle BCD$.

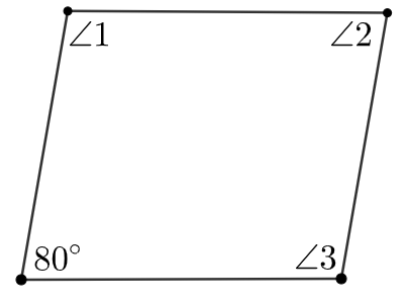


Theorem

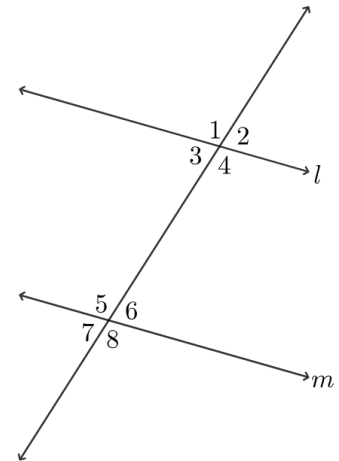
Exterior Angle Theorem: the measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles

Practice

1. The quadrilateral below is a parallelogram. Find $m\angle 1$, $m\angle 2$, $m\angle 3$.



2. In the figure, $l \parallel m$. Identify **all** angles that are congruent to $\angle 2$.



3. In $\triangle ABC$, \overline{AC} is extended through C to D . If $m\angle BAC = (8x - 21)^\circ$, $m\angle ABC = (5x + 1)^\circ$, and $m\angle DCB = (7x + 28)^\circ$, what is the value of $m\angle ACB$?

