Name: ____________________

Lesson 2.04
SAS Congruence

Geometry GT

Recall

A triangle has an angle measuring $35°$ and a side adjacent to the angle with a length of 3cm. Can you create two triangles with those measurements that are **not** congruent?

Explore

Two triangles have two pairs of corresponding sides congruent, and the corresponding angles between those sides (known as an included angle) are congruent. Sketch two triangles, ΔLMN and ΔPQR, such that $\overline{LM} \cong \overline{PQ}$, $\overline{LN} \cong \overline{PR}$, and $\angle L \cong \angle P$.

Use a sequence of rigid motions to take ΔLMN onto ΔPQR. Draw each image in a different color.

Try to draw a third triangle, ΔXYZ, that is **not** congruent to ΔLMN and ΔPQR.

Theorem

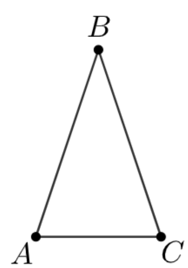
Side-Angle-Side Triangle Congruence Theorem: in two triangles, if two pairs of corresponding sides are congruent, and the corresponding pair of included angles are congruent, then the two triangles are congruent

Definitions

Isosceles triangle: a triangle with two congruent sides
Auxiliary line: an additional line drawn to reveal information

Discuss

Prove that if a triangle is isosceles, then the base angles are congruent. Starting with an isosceles triangle, draw an auxiliary line to create two triangles and use the SAS Triangle Congruence Theorem.



Theorem

Isosceles Triangle Theorem: in an isosceles triangle, the angles opposite the congruent sides are congruent

Demonstrate

It follows from the Side-Angle-Side Triangle Congruence Theorem that if the length of two sides of a triangle are known, and the measure of the included angle is known, then there can only be one possible length for the third side.

Suppose a triangle has sides of lengths 5cm and 12cm. What is the longest the third side could be? What is the shortest it could be?

Practice

1. Malachi is attempting to prove that quadrilateral $ABCD$ is a parallelogram. He knows that $\overline{AB} \cong \overline{DC}$ and $\angle ABC \cong \angle ADC$. Can he use the Side-Angle-Side Triangle Congruence Theorem to say that $\Delta ABC \cong \Delta ADC$ since $\overline{AC} \cong \overline{AC}$? Why or why not?

2. Conjecture: if a point is on the perpendicular bisector of a segment, then that point must be equidistant from the endpoints of the segment.

> **A.** Sketch and label a diagram of the situation. Mark any information you know, such as segments and angles being congruent.
>
> **B.** Find (or add auxiliary lines to find) two triangles that appear congruent. Shade them in using different colors.
>
> **C.** Do you have enough information to prove that the two triangles are congruent? Explain.

3. For each of the figures below, identify what additional information you would need to use the Side-Angle-Side Triangle Congruence Theorem.