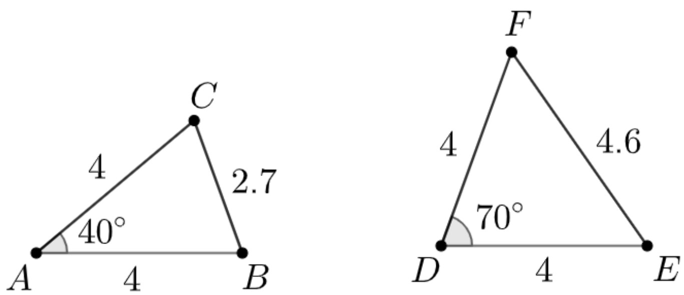
Name: ____________________

Lesson 2.05
AAS/ASA Congruence

Geometry GT

Analyze

What do you notice about the angles and sides in the two triangles below? What do you wonder?



Theorem

In a triangle, the greater side is opposite the greater angle. Conversely, the greater angle is opposite the greater side.

Explore

Triangle ΔABC has the following characteristics:

- $m\angle A = 30°$
- $m\angle B = 95°$
- $AB = 90$ mm
- $BC = 55$ mm
- $AC = 109.5$ mm

Construct triangle ΔABC with dry pasta. Then, attempt to construct a triangle with a longer $\overline{AC}$, and another triangle with a shorter $\overline{AC}$. What happens to the angles and sides of the triangles?

Theorem

Angle-Angle-Side Triangle Congruence Theorem: in two triangles, if two pairs of corresponding angles are congruent, and a corresponding pair of non-included sides are congruent, then the two triangles are congruent

Discuss

Two triangles, ΔWXY and ΔDEF, have two pairs of corresponding angles congruent ($\angle W \cong \angle D$ and $\angle X \cong \angle E$), and the corresponding sides between those angles are congruent ($\overline{WX} \cong \overline{DE}$).
Sketch these two triangles, and use a sequence of rigid motions to take ΔWXY to ΔDEF. Consider how you know that the vertices must line up.

Theorem

Angle-Side-Angle Triangle Congruence Theorem: in two triangles, if two pairs of corresponding angles are congruent, and the corresponding pair of included sides are congruent, then the two triangles are congruent

Demonstrate

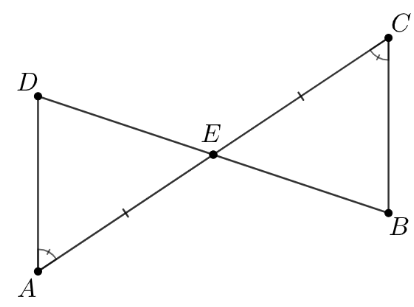
Prove that if a point C is the same distance from A as it is from B, then C must be on the perpendicular bisector of $\overline{AB}$. Hint: sketch the scenario, then consider what auxiliary lines will assist.

Theorem

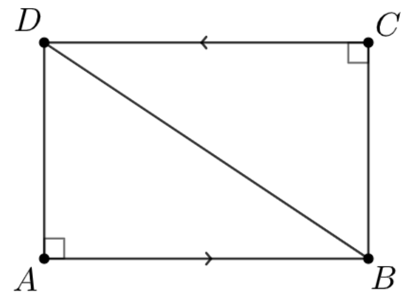
Perpendicular Bisector Theorem: if a point is equidistant from the endpoints of a segment, then it must be on the perpendicular bisector of the segment

Practice

1. What triangle congruence theorem could you use to prove $\Delta ADE \cong \Delta CBE$?



2. Esther wrote a proof that $\Delta BCD \cong \Delta DAB$, but it is incomplete. How can Esther fix her proof?



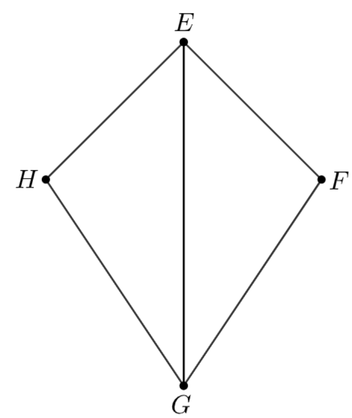
A. Line $\overleftrightarrow{AB}$ is parallel to line $\overleftrightarrow{DC}$ and cut by transversal $\overline{BD}$. So angles $\angle CDB$ and $\angle ABD$ are alternate interior angles and must be congruent.

B. Side $\overline{DB}$ is congruent to side $\overline{BD}$ because they're the same segment.

C. $\angle A$ is congruent to $\angle C$ because they're both right angles.

D. By the Angle-Side-Angle Triangle Congruence Theorem, ΔBCD is congruent to ΔDAB.

3. Segment $\overline{GE}$ is an angle bisector of both $\angle HEF$ and $\angle FGH$. Prove that ΔHGE is congruent to ΔFGE.



4. Triangles ΔACD and ΔBCD are isosceles. If $m\angle BAC = 33°$ and $m\angle BDC = 35°$, find $m\angle ABD$.