Name: ____________________

Lesson 1.01
Construction Basics

Geometry GT

Experiment

In order to gain familiarity with the compass and straightedge, practice drawing multiple lines and circles.

Try to follow these steps:

- Draw a point and label it A
- Draw a circle centered at A
- Mark a point on the circle and label it B
- Draw a circle centered at B and going through A
- Draw segment $\overline{AB}$

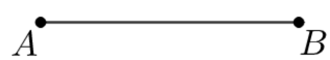
Definitions

Line segment: a set of points on a line with two endpoints
Circle: a set of all points that are the same distance (radius) from a given point (center)

Explore

Given segment $\overline{AB}$, follow these steps:

- Draw a circle centered at A with radius AB
- Mark a point at the middle of $\overline{AB}$ and label it C
- Draw a circle centered at B with radius BC
- Label the intersection of the circles above B as D and below B as E
- Draw segments $\overline{AD}$, $\overline{DE}$, and $\overline{AE}$
- Trace ΔADE onto patty paper



Compare your ΔADE with your neighbors. Why might they be different? How could we ensure they are all the same?

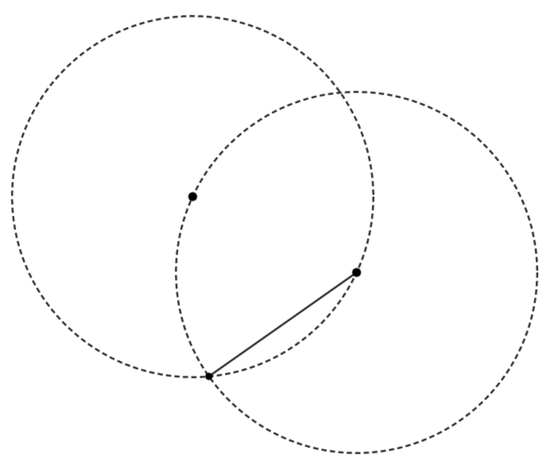
Discuss

Using a straightedge and compass, there are a few basic construction moves that are valid:

- Draw points in blank space, on objects, and at intersections
- Draw segments, rays, and lines through two points using a straightedge
- Draw a circle centered at a point and through another point using a compass
- Set the compass to a length between two points then move the compass (preserving the length)

Demonstrate

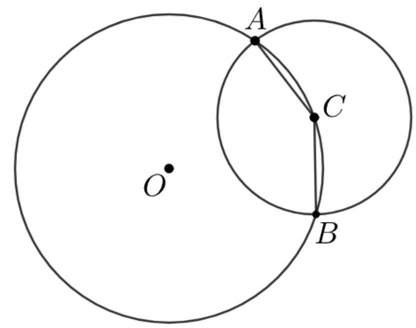
The figure shows the first few steps of constructing a regular hexagon. Complete the construction.



A **regular polygon** has sides with equal lengths. How can you be sure your hexagon is a regular hexagon?

Practice

1. Here is a diagram of a straightedge and compass construction. O is the center of one circle, and C is the center of the other. Explain why the length of segment $\overline{AC}$ is the same as the length of segment $\overline{BC}$.



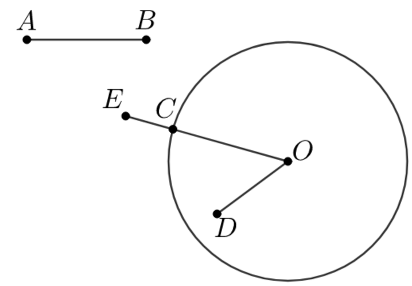
2. Esther used a compass to make a circle with radius the same length as segment $\overline{AB}$. She labeled the center O. Which statement is true?

A. $AB > OD$

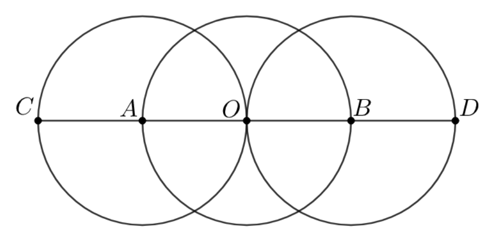
B. $AB = OD$

C. $AB > OE$

D. $AB = OE$



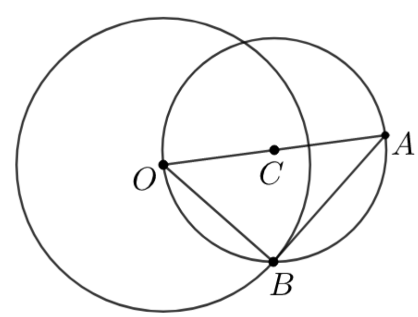
3. The diagram was constructed with straightedge and compass tools. Points A, B, C, D, and O are all on line segment $\overline{CD}$. Name a line segment that is half the length of $\overline{CD}$. Explain how you know.



4. This diagram was constructed with straightedge nad compass tools. O is the center of one circle, and C is the center of the other.

A. The 2 circles intersect at point B. Label the other intersection point D.

B. How does the length of segment $\overline{CD}$ compare to the length of segment $\overline{OA}$?



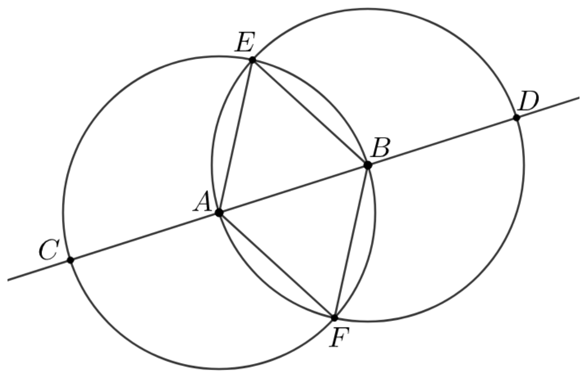
Name: ____________________

Lesson 1.02
Patterns & Instructions

Geometry GT

Recall

Here are 2 circles with centers A and B.



Based on the diagram, explain how you know each statement is true.

A. The length of segment $\overline{AE}$ is equal to the length of segment $\overline{BE}$

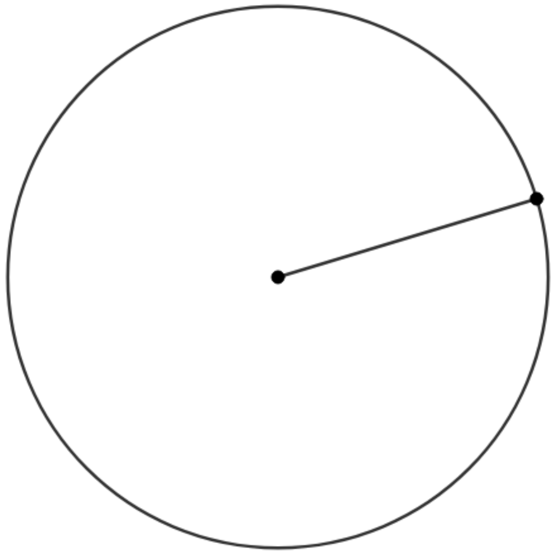
B. ΔABF is equilateral

C. $AB = \dfrac{1}{3}CD$

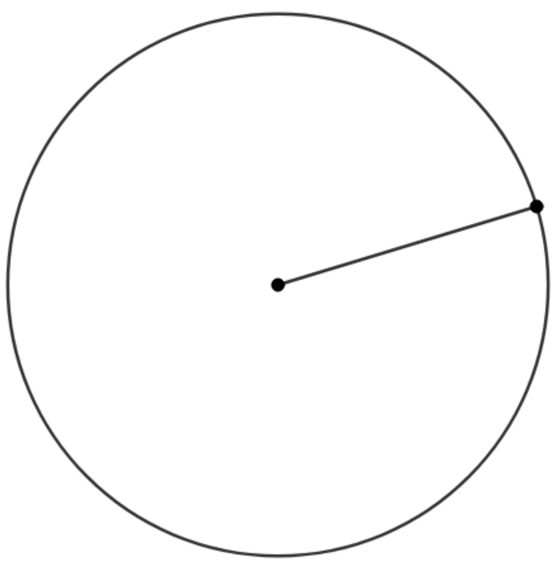
D. $BC = AD$

Explore

Use straightedge and compass moves to build your own pattern using the circle and radius as a place to start. As you make your pattern, record each move on a separate sheet of paper. Use precise vocabulary so someone can make a perfect copy without seeing the original. Include instructions about how to shade/color your pattern.



Follow someone else's instructions precisely to recreate their pattern.



Discuss

What was difficult about following someone's instructions?

What changes would you make about the way you wrote your instructions to describe figures in geometry?

Were there any shapes or patterns that you were surprised could be made with straightedge and compass moves?

Demonstrate

Follow the directions to construct an equilateral triangle.

A. Start with two points

B. Draw circles at both points

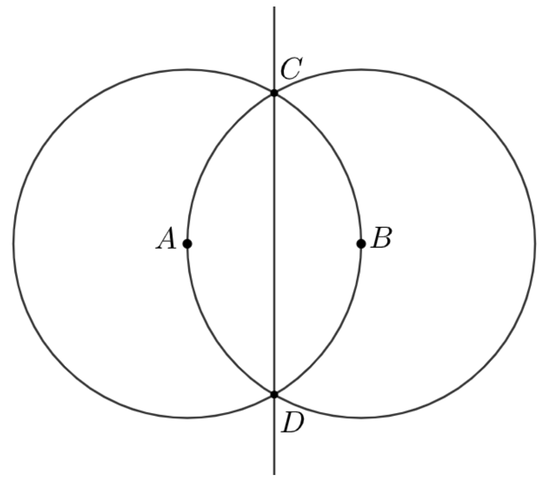
C. Mark a point on both circles

D. Draw lines between the points

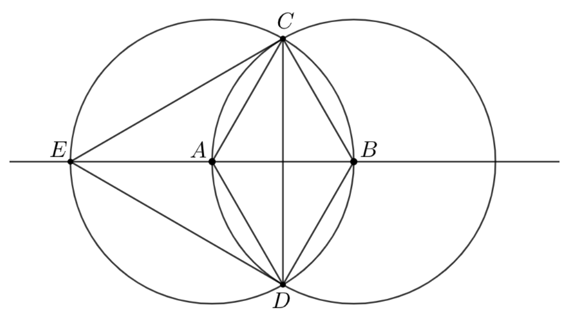
How could these directions be improved? What information would be useful?

Practice

1. This diagram was created by starting with points A and B and using only straightedge and compass to construct the rest. All steps of the construction are visible. Describe precisely the straightedge and compass moves required to construct the line $\overleftrightarrow{CD}$ in this diagram?



2. In the construction, A is the center of one circle, and B is the center of the other. Identify **all** segments that have the same length as segment $\overline{AB}$.



A. $\overline{AC}$

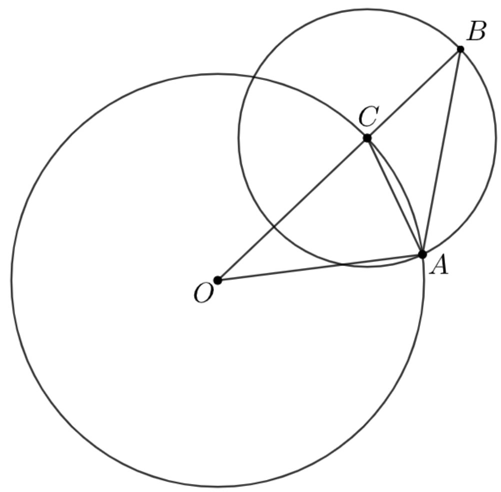
B. $\overline{AE}$

C. $\overline{BC}$

D. $\overline{CD}$

E. $\overline{DE}$

3. This diagram was constructed with straightedge and compass tools. O is the center of one circle, and C is the center of the other. Select **all** line segments that *must* have the same length as segment $\overline{OA}$.



A. $\overline{OA}$

B. $\overline{OC}$

C. $\overline{AC}$

D. $\overline{AB}$

E. $\overline{BC}$

Name: ____________________

Lesson 1.03
Constructing Bisectors

Geometry GT

Experiment

Here are two points labeled A and B, and line segment $\overline{CD}$.

A. Mark 5 points that are a distance CD away from point A. How could you describe all points that are a distance CD away from point A?

B. Mark 5 points that are a distance CD away from point B. How could you describe all points that are a distance CD away from point B?

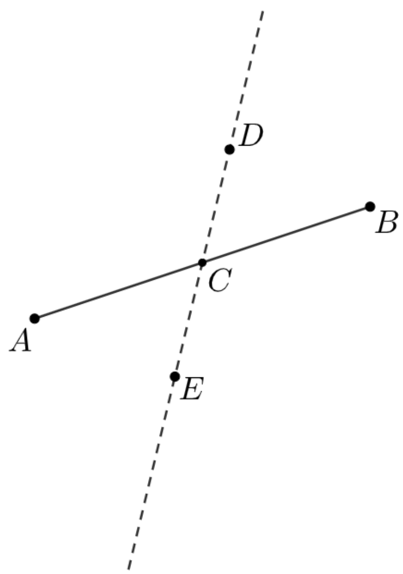
C. In a different color, mark all the points that are a distance CD away from both A and B at the same time.

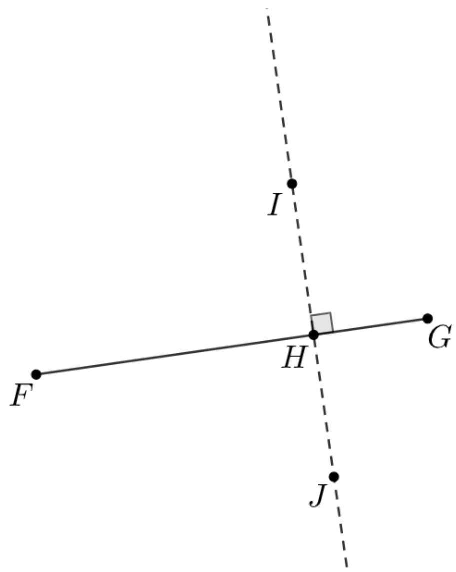
Definitions

Perpendicular lines: two lines that intersect at a point to create right angles
Perpendicular bisector: a line through the midpoint of a segment that is perpendicular to the segment

Explore

Examine the two figures below.





Explain why each dashed line is *not* a perpendicular bisector of the segment it intersects.

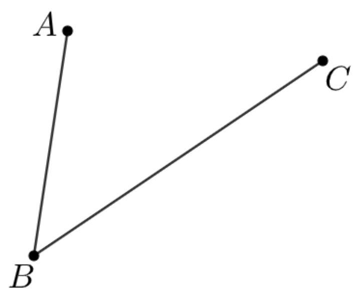
Use compass and straightedge moves to construct the perpendicular bisector of segment $\overline{PQ}$.

Definition

Angle bisector: a line through the vertex of an angle that divides it into two equal angles

Discuss

Here is angle $\angle ABC$.



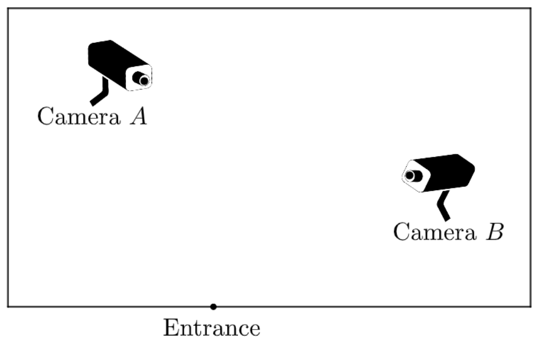
Use compass and straightedge moves to construct a ray that divides $\angle ABC$ into 2 congruent angles. Then, on another sheet of paper, draw another angle and have your neighbor attempt to bisect it.

Write precise instructions for constructing a perpendicular bisector and angle bisector below.

Perpendicular bisector | Angle bisector

Demonstrate

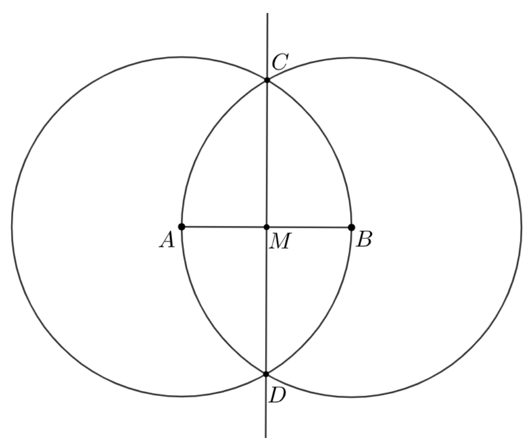
Scenario: You are attempting to covertly sneak through a secure room. There are two security cameras mounted to the ceiling, and they start recording whenever a moving object is closer to one that the other. However, due to some lazy programming, if a moving object is equidistant from both cameras, neither will start recording.



Using a straightedge and compass, map out the path you could take to cross the room without being caught on camera.

Practice

1. This diagram is a straightedge and compass construction. A is the center of one circle, and B is the center of the other. Select **all** the true statements.



A. Line $\overleftrightarrow{CD}$ is perpendicular to segment $\overline{AB}$

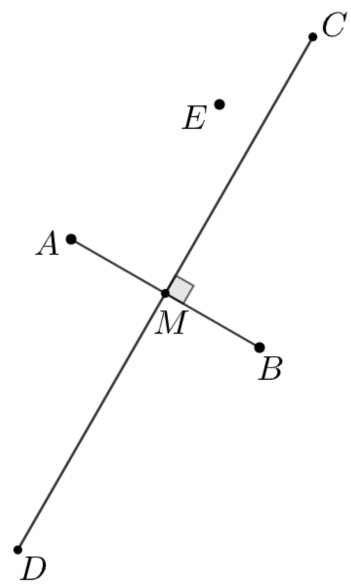
B. Point M is the midpoint of segment $\overline{AB}$

C. The length AB is equal to the length CD

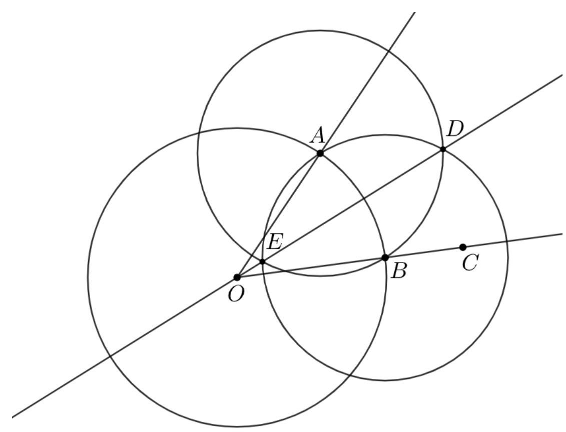
D. Segment $\overline{AM}$ is perpendicular to segment $\overline{BM}$

E. $CB + BD > CD$

2. In this diagram, line segment $\overline{CD}$ is the perpendicular bisector of the line segment $\overline{AB}$. Assume the conjecture that the set of points equidistant from A and B is the perpendicular bisector of $\overline{AB}$ is true. Is point E closer to point A, closer to point B, or the same distance between the points? Explain how you know.



3. This diagram is a straightedge and compass construction. Select **all** true statements.



A. Line $\overleftrightarrow{DE}$ is the bisector of $\angle AOC$

B. Line $\overleftrightarrow{DE}$ is the perpendicular bisector of segment $\overline{AO}$

C. Line $\overleftrightarrow{DE}$ is the perpendicular bisector of segment $\overline{CO}$

D. Line $\overleftrightarrow{DE}$ is the perpendicular bisector of segment $\overline{AB}$

E. Line $\overleftrightarrow{DE}$ is parallel to line $\overleftrightarrow{BC}$

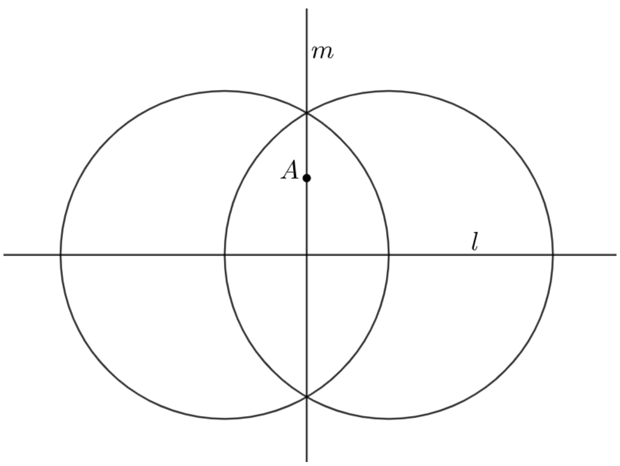
Name: ______________________________

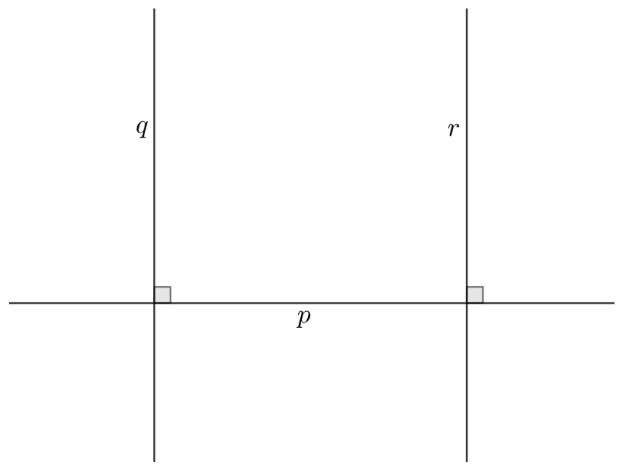
Lesson 1.04
Constructing Lines

Geometry GT

Analyze

Consider the two figures below.

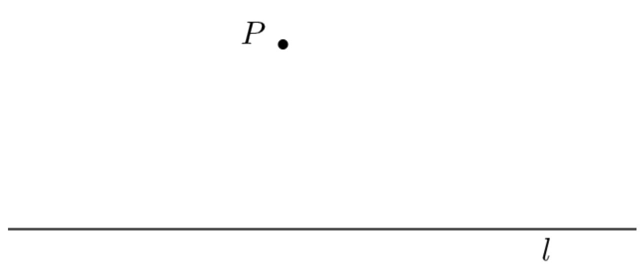




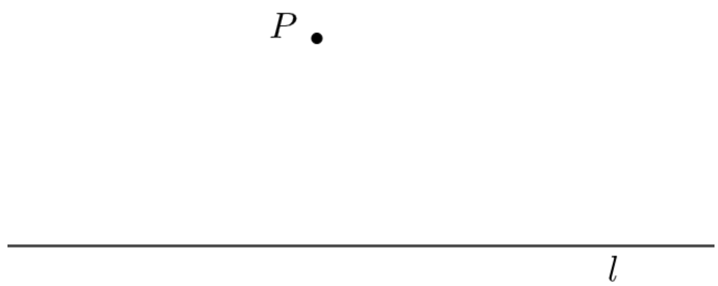
What do you notice about each figure? What do you wonder?

Explore

How does the following figure differ from the starting figure for creating a perpendicular bisector?



Use compass and straightedge moves to construct a line that is perpendicular to line l and goes through point P. You may extend line l if necessary.



Discuss

How would the construction change if point P was on line l?

Use compass and straightedge moves to construct a line that is perpendicular to line l and goes through point P. You may extend line l if necessary.

To the right, write precise instructions for constructing a line that is perpendicular to a given line and goes through a specific point.

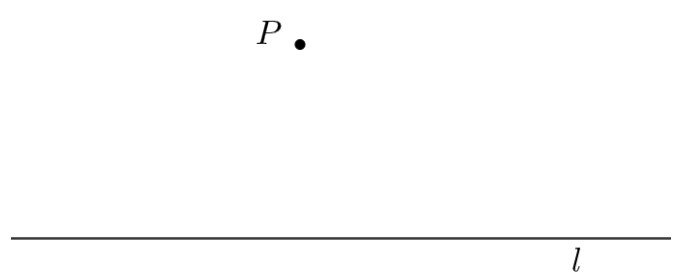
Perpendicular line through a point

Definition

Parallel lines: two lines that do not intersect at any point

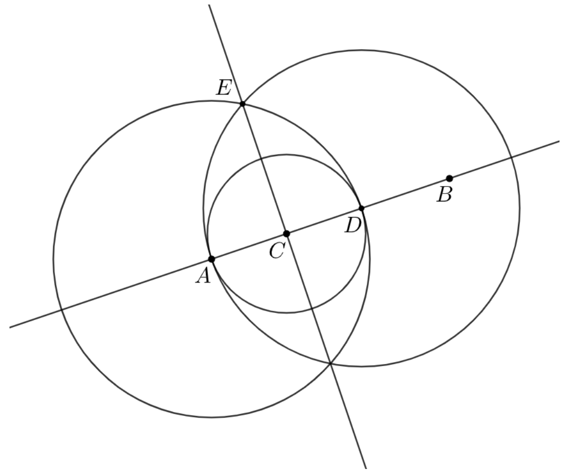
Demonstrate

Use compass and straightedge moves to construct a line that is *parallel* to line l and goes through point P. You may extend line l if necessary.



Practice

1. This diagram is a straightedge and compass construction of a line perpendicular to line $\overleftrightarrow{AB}$ passing through point C.



A. Which segment has the same length as segment $\overline{AE}$?

B. Explain why it was helpful to construct points D and A to be the same distance from C.

2. Two distinct lines, l and m, are each perpendicular to the same line n. Select **all** the true statements.

A. $l \perp m$

B. $l \perp n$

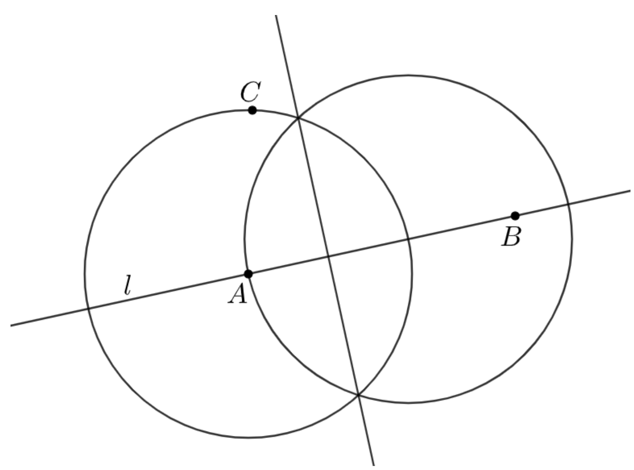
C. $m \perp n$

D. $l \parallel m$

E. $l \parallel n$

F. $m \parallel n$

3. Siena wanted to construct a line perpendicular to line l through point C. The diagram shows her construction. What was her mistake?



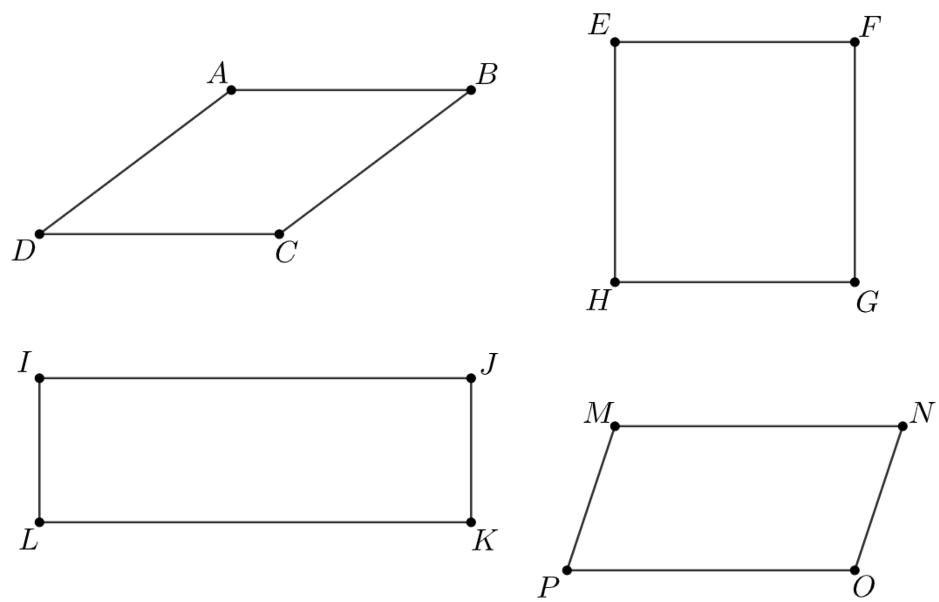
Name: ________________________________

Lesson 1.05
Squares

Geometry GT

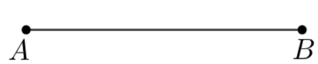
Recall

For each figure below, determine what type of quadrilateral it is.



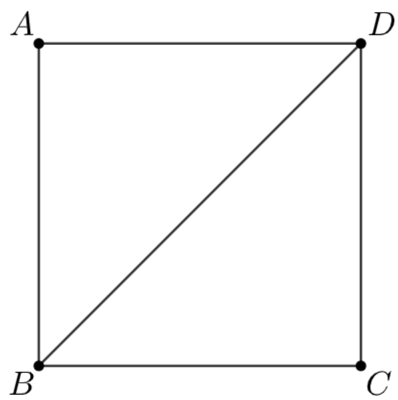
Explore

Use straightedge and compass moves to construct a square with segment $\overline{AB}$ as one of the sides.



How do you know that what you constructed is a square?

Here is square $ABCD$ with diagonal $\overline{BD}$ drawn.

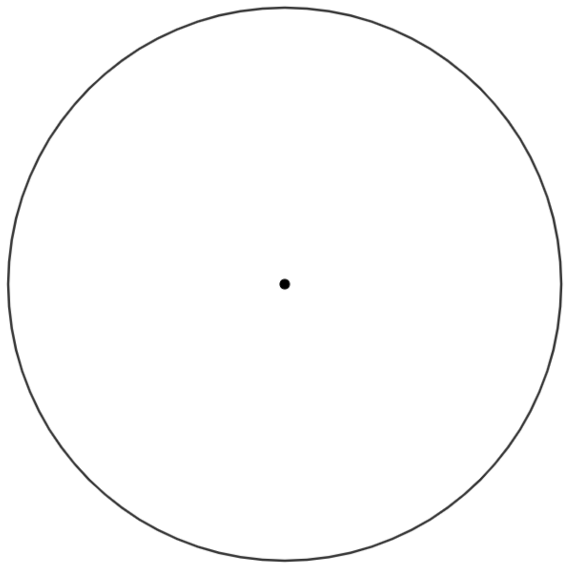


First, construct a circle centered at A with radius AD and another circle centered at C with radius CD. Then draw the diagonal $\overline{AC}$. Write a conjecture about the relationship between diagonals $\overline{BD}$ and $\overline{AC}$.

Label the intersection of the diagonals as point E and construct a circle centered at E with radius BE. How are the diagonals related to this circle?

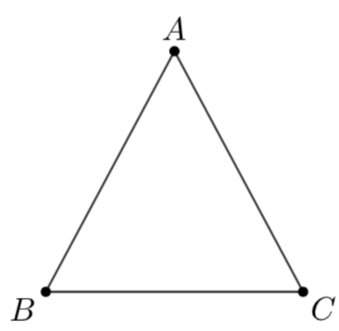
Discuss

Use straightedge and compass moves to construct a square inscribed in a circle.



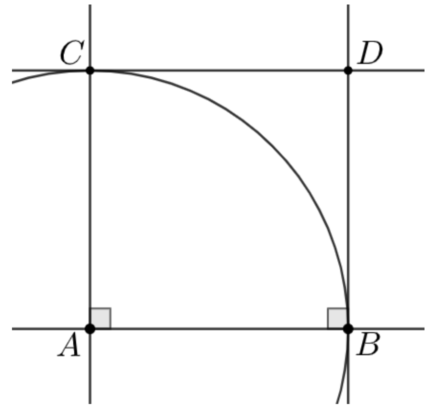
Demonstrate

Use compass and straightedge moves to construct a square with segment $\overline{BC}$ as one of the sides.



Practice

1. This diagram is a straightedge and compass construction of a square $BACD$ (not all markings are shown). The construction followed these steps:



A. Start with two marked points A and B

B. Use a straightedge to construct line $\overleftrightarrow{AB}$

C. Use a previous construction to construct a line perpendicular to $\overleftrightarrow{AB}$ passing through A

D. Use a previous construction to construct a line perpendicular to $\overleftrightarrow{AB}$ passing through B

E. Use a compass to construct a circle centered at A passing through B

F. Label an intersection point of that circle and the line from step **C** as C

G. Use a previous construction to construct a line parallel to $\overleftrightarrow{AB}$ passing through C

H. Label the intersection of that line and the line from step **D** as D

I. Use a straightedge to construct the segments $\overline{AC}$, $\overline{CD}$, and $\overline{BD}$

Explain why you need to construct a circle in step **E**.

2. Which of these statements is true?

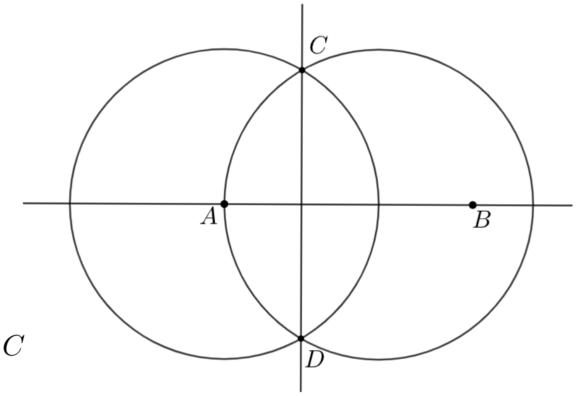
A. All rectangles are regular polygons

B. All squares are regular polygons

C. All rhombi are regular polygons

D. All parallelograms are regular polygons

3. To construct a line passing through the point C that is parallel to the line $\overleftrightarrow{AB}$, the first step is to create a line through C perpendicular to $\overleftrightarrow{AB}$. What is the next step?



A. Construct an equilateral triangle with side $\overline{CD}$

B. Construct a line through point B perpendicular to $\overleftrightarrow{AB}$

C. Construct a segment with length AB with endpoint C

D. Construct a line through point C perpendicular to $\overleftrightarrow{CD}$

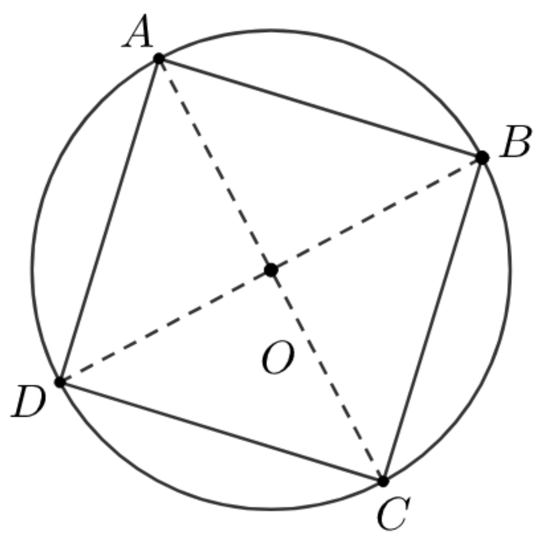
Name: ____________________

Lesson 1.06
Deep Dive: Constructions

Geometry GT

Task #1

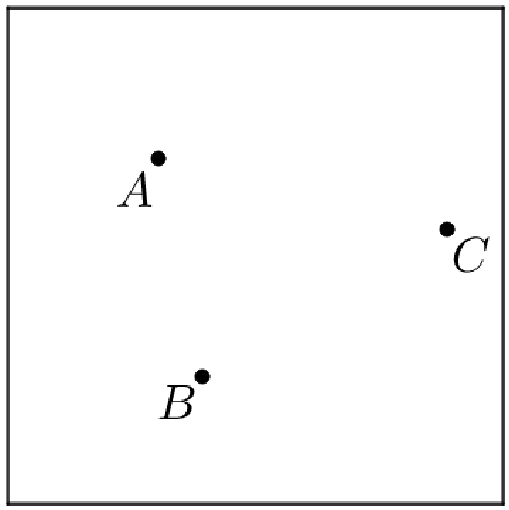
The figure below is a square inscribed in a circle. Use straightedge and compass moves to construct a square that fits perfectly outside the circle, so that the circle is inscribed in the square.



Write precise instructions for constructing a square that fits outside the circle.

Task #2

SCENARIO: The Delectable Diner has three locations in Square City. The owner wants to divide the city into three regions so that whenever an online order is placed, it is sent to the store closest to the customer.



Partition the city into the three regions using a straightedge and compass. Color the three regions with different colors. How do you know your partitioning is accurate?

If all there are 100 total employees working for The Delectable Diner, estimate how they should be distributed between the three locations.

Is there any spot in the city that has the same distance from all three stores?

Areas for Improvement	Standards and Criteria	Areas of Excellence
	Reason abstractly and quantitatively Describe why certain geometric concepts are true and apply the ideas to specific scenarios.	
	Use appropriate tools strategically Accurately use a straightedge and compass to construct the appropriate figures.	
	Attend to precision Use precise mathematical language in writing instructions and utilize accurate definitions.	

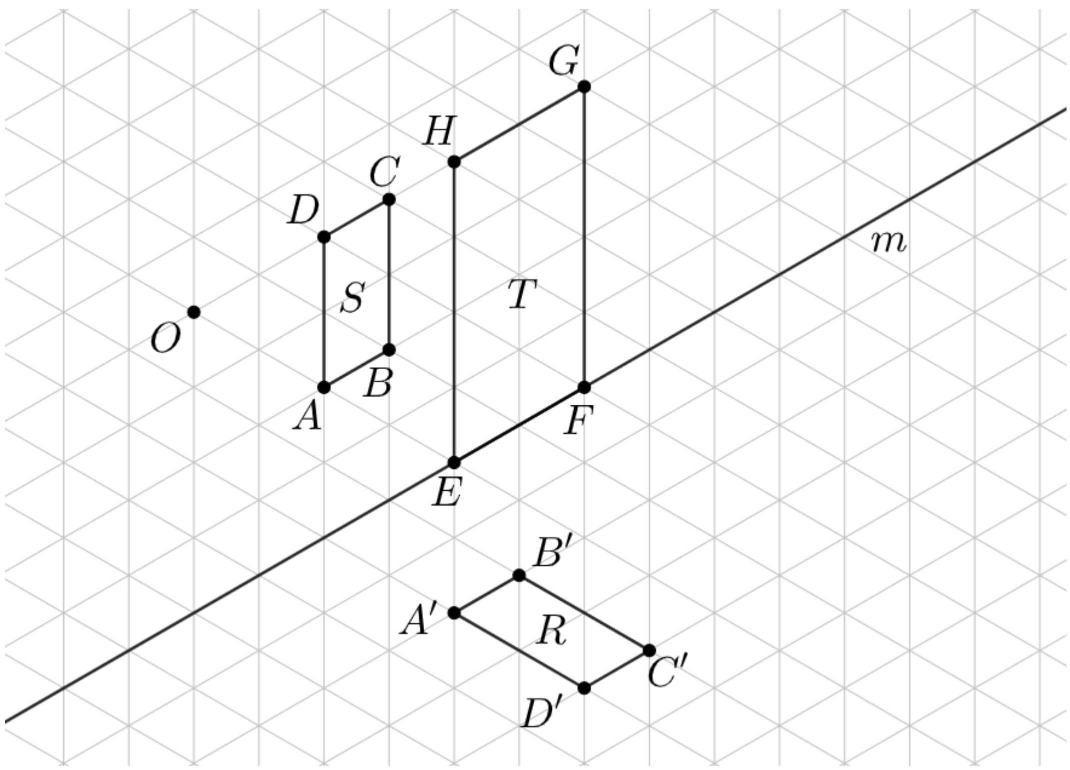
Name: ____________________

Lesson 1.07
Rigid Motion

Geometry GT

Analyze

What do you notice? What do you wonder?

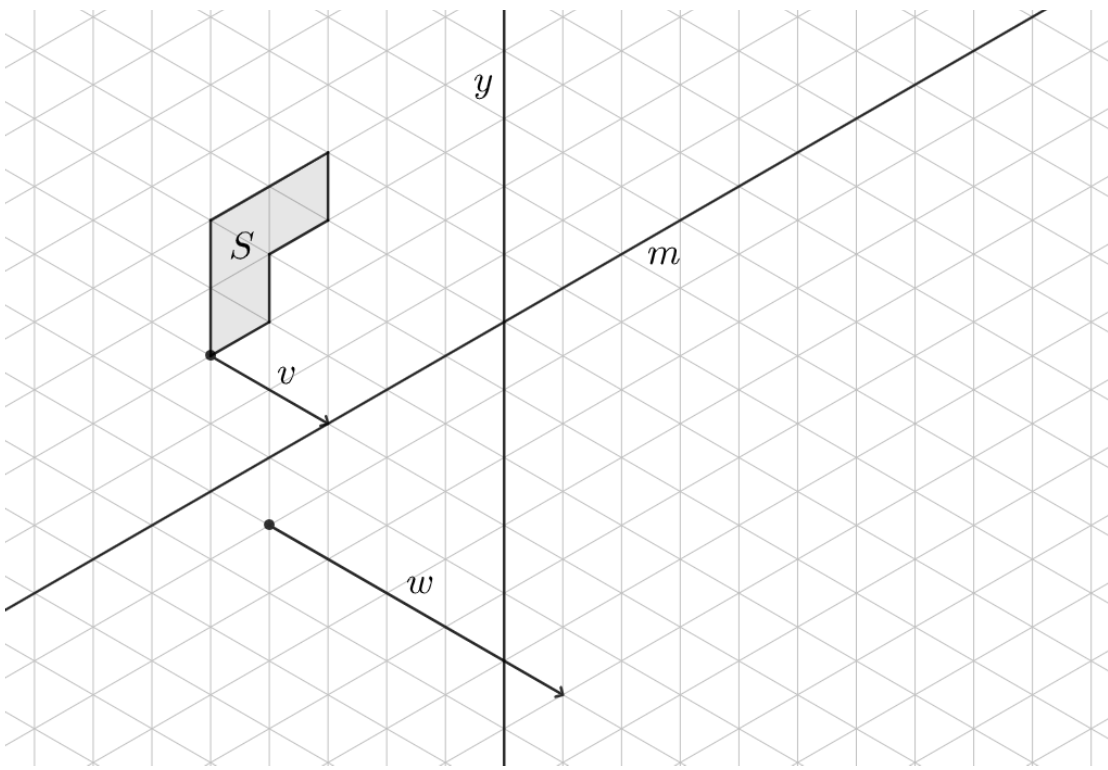


Explore

In your own words, what is the definition of a **translation**?

In your own words, what is the definition of a **reflection**?

Draw each rigid transformation in a different color.



A. Translate figure S along the line segment v in the direction shown by the arrow.

B. Reflect figure S across line y.

C. Reflect figure S across line m.

D. Translate figure S along the line segment w in the direction shown by the arrow.

E. Reflect the image from **D** across line y.

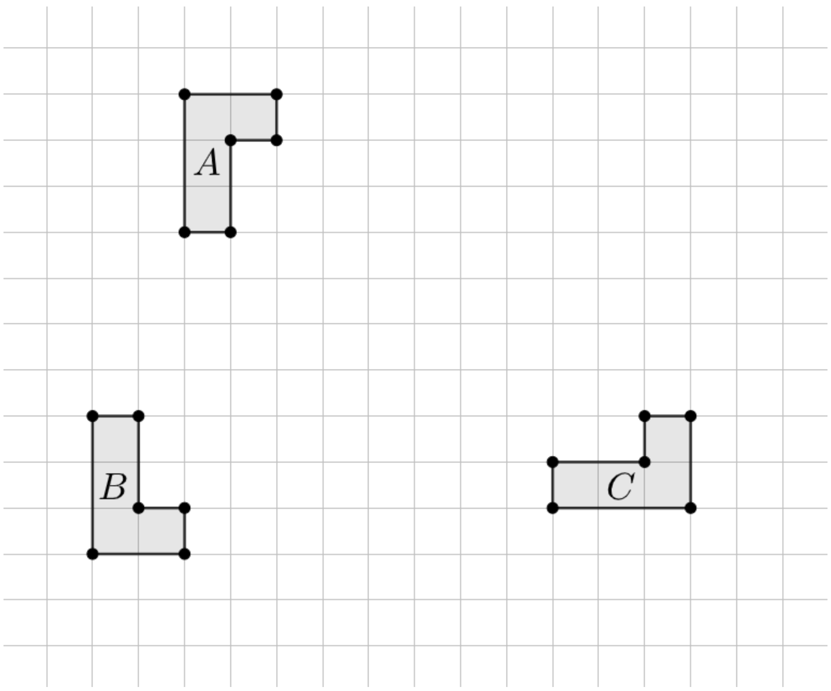
F. How are the images the same? How are they different?

Definitions

Rigid transformation: a translation, rotation, or reflection of a figure; sometimes used to refer to a sequence of transformations
Image: the new position of a figure after a transformation is applied
Congruent: two figures with the same shape and size; if there exists a rigid transformation that takes one figure onto another, then the two figures are congruent

Discuss

Here are 3 congruent L-shapes.



A. Describe a sequence of transformations that will take Figure A onto Figure B.

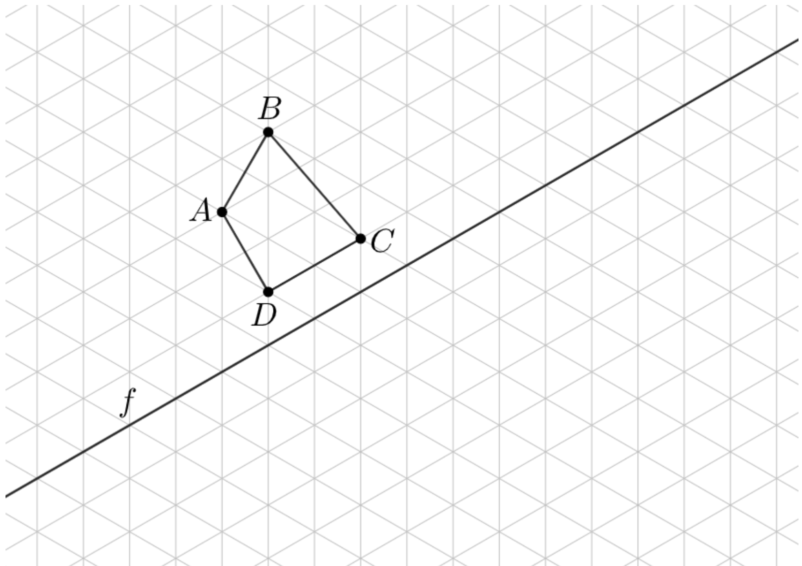
B. If you reverse the order of your sequence, will the reverse sequence still take A onto B?

C. Describe a sequence of transformations that will take Figure A onto Figure C.

D. If you reverse the order of your sequence, will the reverse sequence still take A onto C?

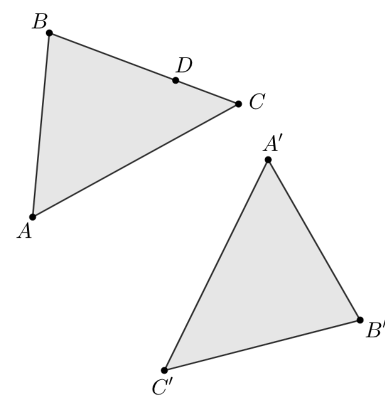
Demonstrate

Reflect quadrilateral $ABCD$ across line f.



Practice

1. There is a sequence of rigid transformations that takes A to A', B to B', and C to C'. The same sequence takes D to D'. Draw and label D'.



2. Which construction could be used to construct an isosceles triangle ΔABC given line segment $\overline{AB}$?

A. Mark a third point C not on segment $\overline{AB}$. Draw segments $\overline{AC}$ and $\overline{BC}$.

B. Label a point C on segment $\overline{AB}$ and construct a line perpendicular to $\overline{AB}$ through point C. Draw segments $\overline{AC}$ and $\overline{BC}$.

C. Construct the perpendicular bisector of segment $\overline{AB}$. Mark the intersection of this line and $\overline{AB}$ and label it C. Draw segments $\overline{AC}$ and $\overline{BC}$.

D. Construct the perpendicular bisector of segment $\overline{AB}$. Mark any point C on the perpendicular bisector except where it intersects $\overline{AB}$. Draw segments $\overline{AC}$ and $\overline{BC}$.

3. Select **all** true statements about regular polygons.

A. All angles are right angles

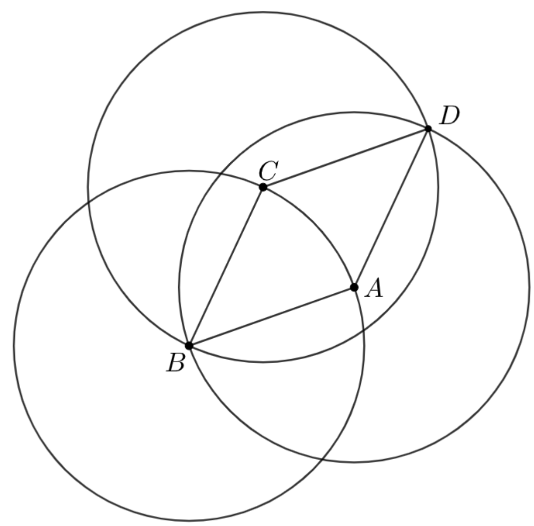
B. All angles are congruent

C. All side lengths are equal

D. There are exactly 4 sides

E. There are at least 3 sides

4. This straightedge and compass construction shows quadrilateral $ABCD$. Is $ABCD$ a rhombus? Explain how you know.



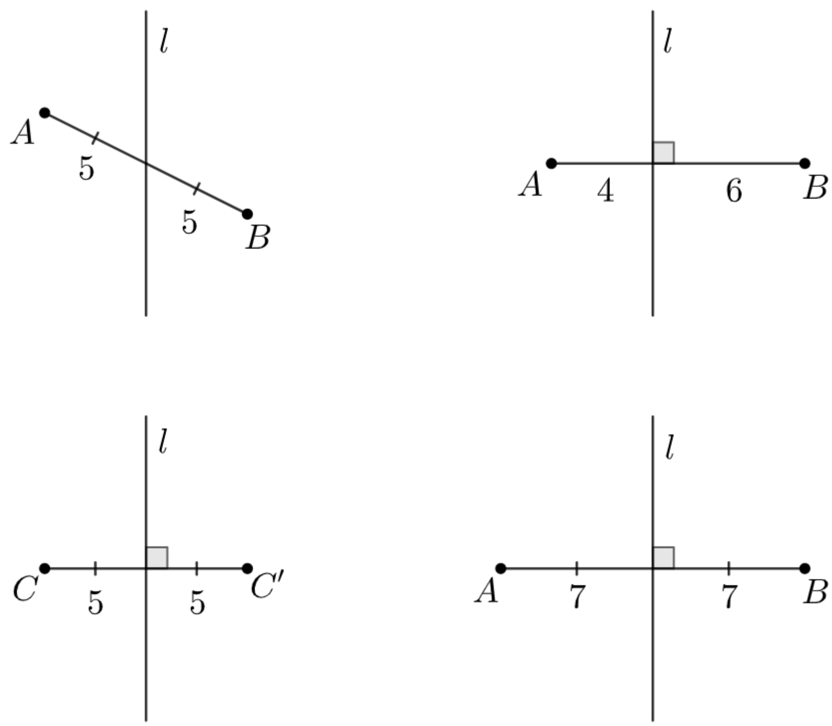
Name: ______________________

Lesson 1.08
Reflections

Geometry GT

Analyze

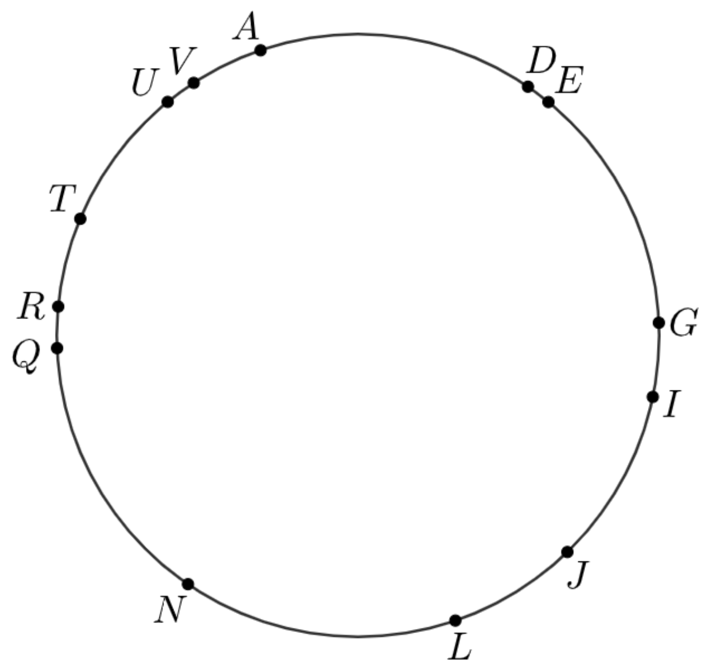
Which one doesn't belong?



First, think to yourself, then share your choice and reasoning with a neighbor. Write any notes below.

Explore

DIRECTIONS: You and a partner will each receive a data card for a different scenario (do **not** show them to each other). For each scenario, the person *without* the data card will ask their partner for information to help solve the problem. Consider what information you need and why you need it.

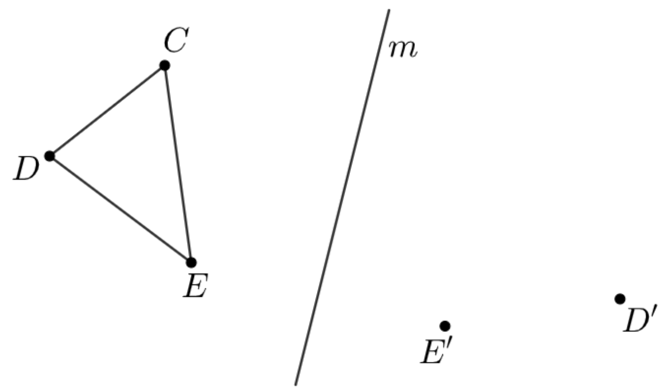


SCENARIO A: Triangle ΔGEN has been reflected so that the vertices of its image are labeled points. What is the image of ΔGEN?

SCENARIO B: Several points have been reflected across a line that goes through 2 of the labeled points. Precisely describe the reflection.

Discuss

Cam started reflecting triangle ΔCDE across line m. So far, he knows the image of D is D' and the image of E is E'.



A. Annotate Cam's diagram to show how he reflected point D.

B. Use straightedge and compass moves to determine the location of C'. Then lightly shade in $\Delta C'D'E'$.

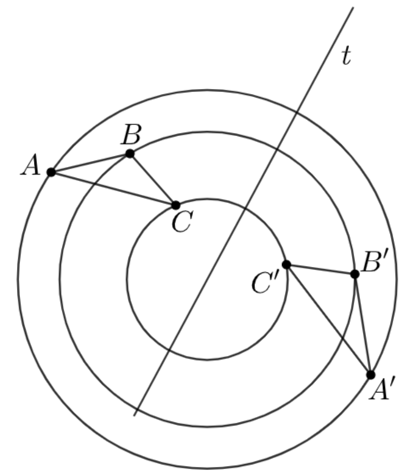
C. Write a set of instructions for how to reflect any point P across a given line l.

Definition

Reflection: a rigid transformation that takes a point to another point that is the same distance from the given line, is on the other side of the given line, and so that the segment from the original point to the image is perpendicular to the given line

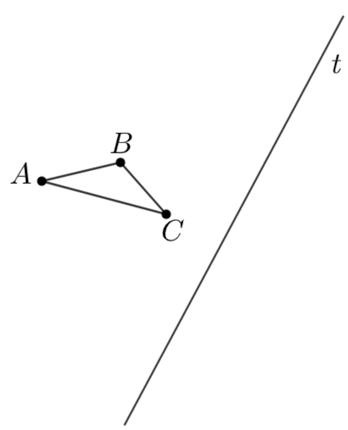
Demonstrate

Krish tried to reflect ΔABC across line t. He knows something went wrong because the image isn't congruent to the original figure.



A. What is one idea that Krish does not understand about reflections?

B. Reflect ΔABC correctly.



Practice

1. Which of these construction would construct a line of reflection that takes point A to point B?

A. Construct the perpendicular bisector of $\overline{AB}$

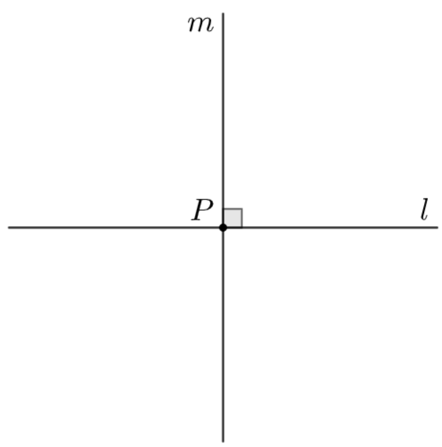
B. Construct a line through B perpendicular to $\overline{AB}$

C. Construct the line passing through A and B

D. Construct a line parallel to $\overleftrightarrow{AB}$

2. A point P stays in the same location when it is reflected over line l. What can you conclude about P?

3. Lines l and m are perpendicular with point of intersection P. Maya says that a 180° rotation, with center P, has the same effect on points in the plane as reflecting over line m. Do you agree with Maya? Explain your reasoning.



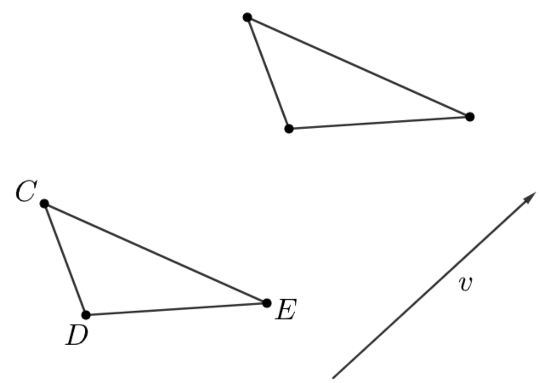
Name: ______________________

Lesson 1.09
Translations

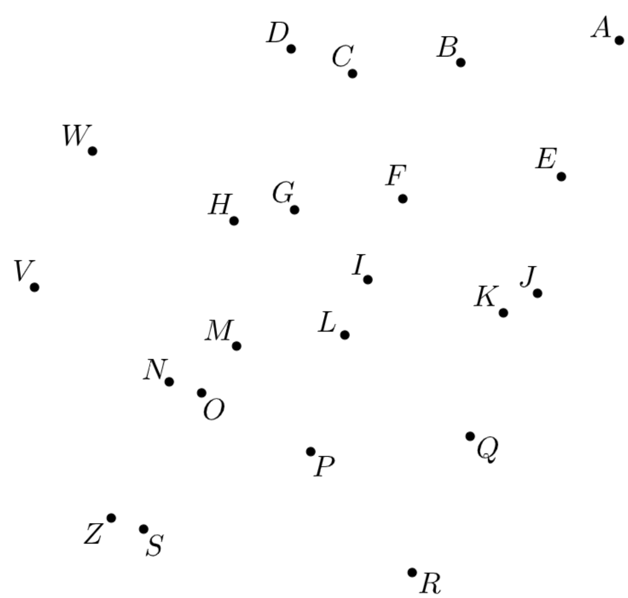
Geometry GT

Analyze

What do you notice? What do you wonder?



Explore



A. After a translation, the image of V is W. Find at least 3 other points that are taken to a labeled point by that translation.

B. Write at least 1 conjecture about translations.

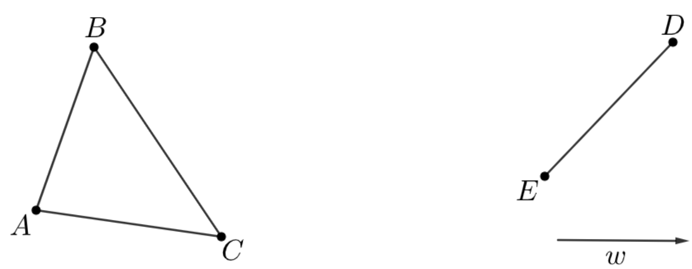
C. In a new translation, the image of V is Z. Find at least 3 other points that are taken to a labeled point by that translation.

D. Are your conjectures still true for the new translation?

Definition

Directed line segment: a line segment with a direction to it; conveys the direction and distance that each point is translated

Discuss



Translate ΔABC by the directed line segment from A to C.

A. What is the relationship between $\overleftrightarrow{BC}$ and $\overleftrightarrow{B'C'}$? Explain your reasoning.

B. How does the length of $\overline{BC}$ compare to the length of $\overline{B'C'}$? Explain your reasoning.

Translate $\overline{DE}$ by the directed line segment w. Label the new endpoints D' and E'.

A. Connect D to D' and E to E'.

B. What kind of shape did you draw? What properties does it have? Explain your reasoning.

Definition

Translation: a rigid transformation that takes a point to another point so that the directed line segment from the original point to the image is parallel to the given line segment and has the same length and direction

Assertion

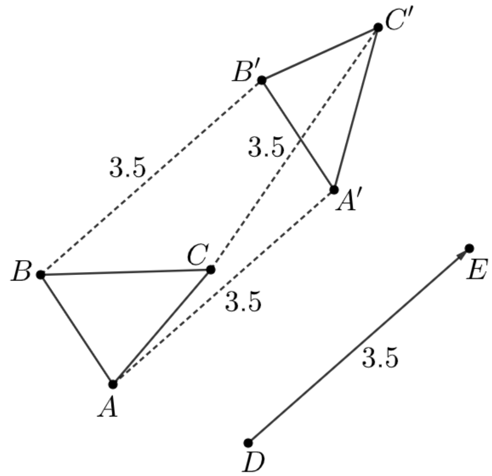
Parallel Postulate: given a line and a point not on the line, there is exactly one line that goes through the point that is parallel to the given line

Theorem

Translations take lines to parallel lines or to themselves

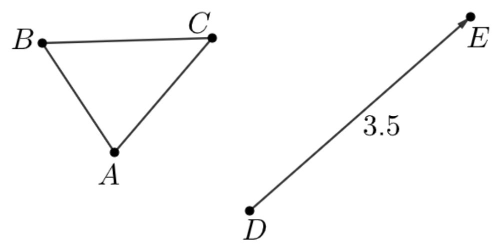
Demonstrate

Noel tried to translate ΔABC by the directed line segment from D to E. He knows something went wrong because the image isn't congruent to the original figure.



A. What is one idea that Noel does not understand about translations?

B. Translate ΔABC correctly.



Practice

1. Which statement is true about a translation?

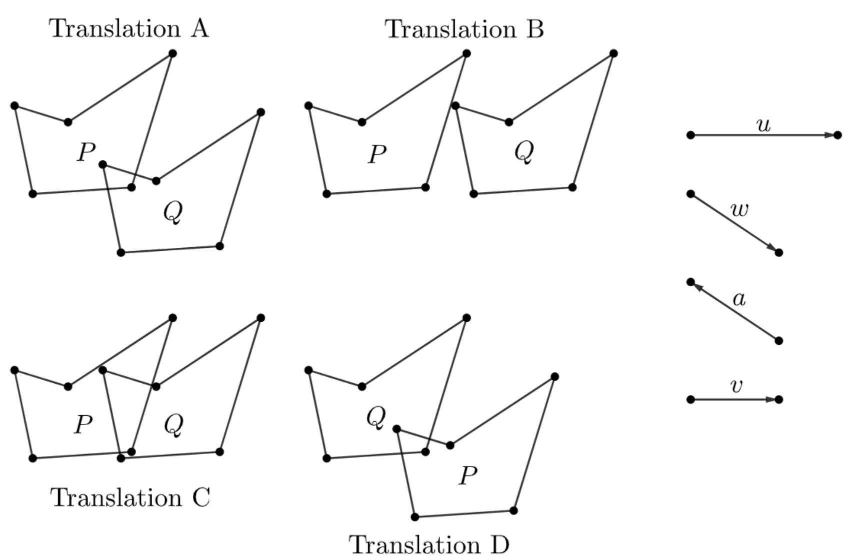
A. A translation takes a line to a parallel line or itself

B. A translation takes a line to a perpendicular line

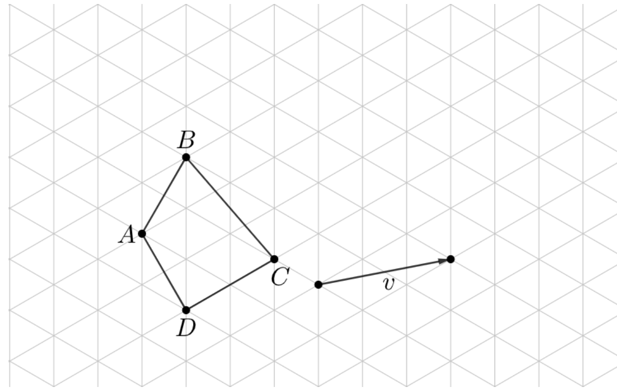
C. A translation requires a center of translation

D. A translation requires a line of translation

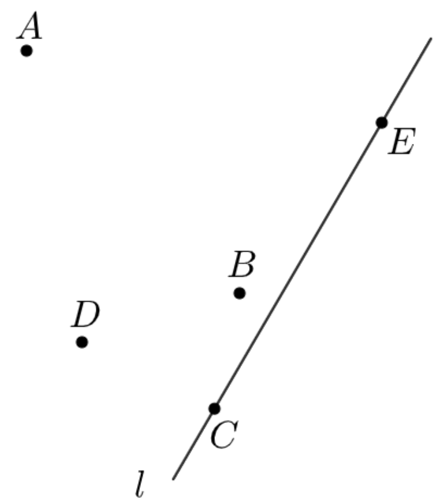
2. Match the directed line segment with the image of Polygon P being transformed to Polygon Q by translation by that directed line segment.



3. Draw the image of quadrilateral $ABCD$ when translated by the directed line segment v. Label the image of A as A', the image of B as B', the image of C as C', and the image of D as D'.



4. Highlight **all** the points that stay in the same location after being reflected across line l.



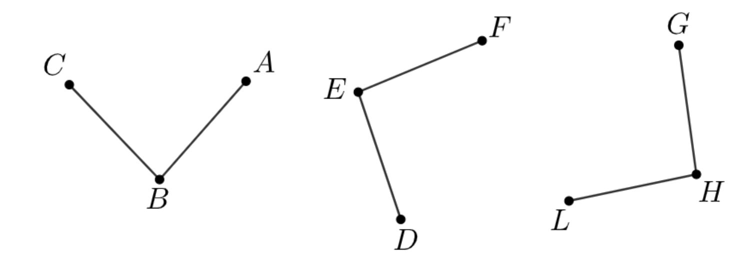
Name: ______________________________

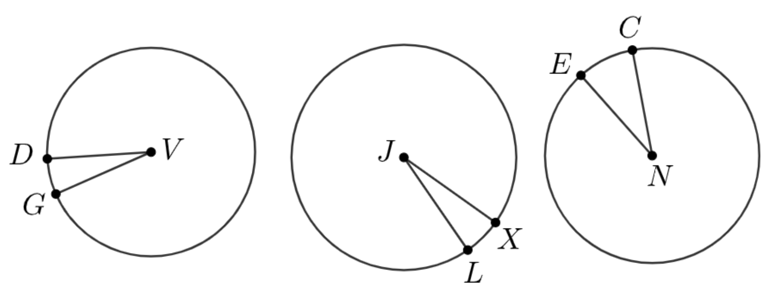
Lesson 1.10
Rotations

Geometry GT

Recall

For each figure, which pair of angles appears congruent? How could you check?





Explore

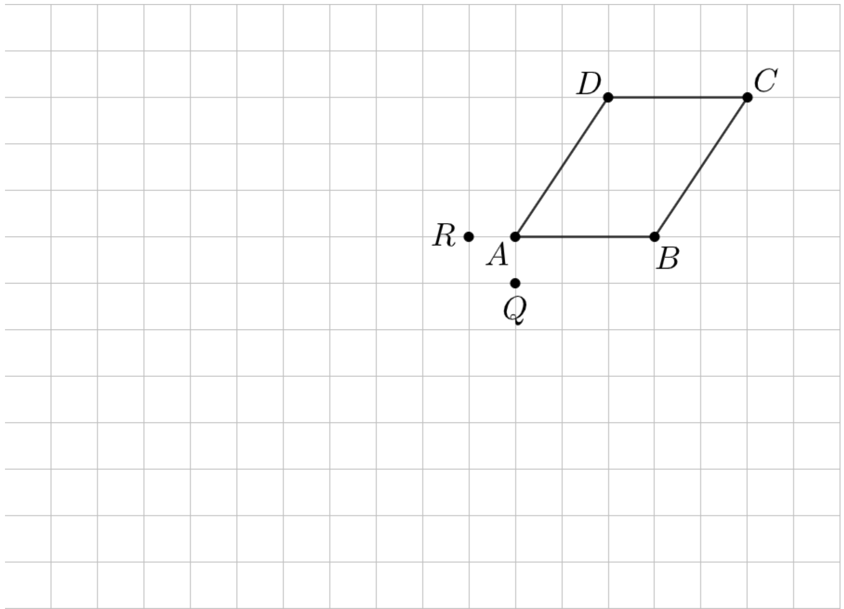
Use the grids to complete the rotations.

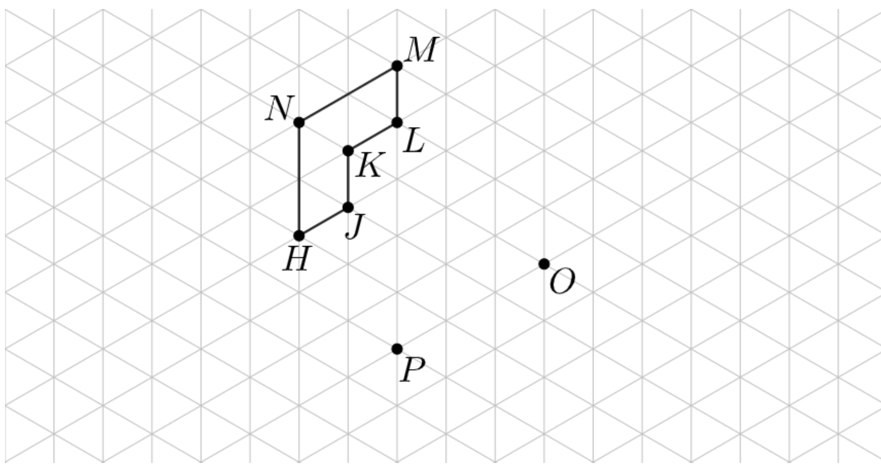
A. Rotate $ABCD$ 90° clockwise around Q.

B. Rotate $ABCD$ 180° around R.

C. Rotate $HJKLMN$ 120° clockwise around O.

D. Rotate $HJKLMN$ 60° counterclockwise around P.





Discuss

Draw a segment. Label the endpoints A and B.

A. Rotate segment $\overline{AB}$ clockwise around center B by $90°$. Label the new endpoint A'.

B. Connect A to A' and lightly shade in the resulting triangle.

C. What kind of triangle did you draw? What other properties do you notice in the figure? Explain your reasoning.

Draw a segment. Label the endpoints C and D.

A. Rotate segment $\overline{CD}$ counterclockwise around center D by 30°. Label the new endpoint C'.

B. Rotate segment $\overline{C'D}$ counterclockwise around center D by 30°. Label the new endpoint C''.

C. Connect C to C'' and lightly shade in the resulting triangle.

D. What kind of triangle did you draw? What other properties do you notice in the figure? Explain your reasoning.

Definition

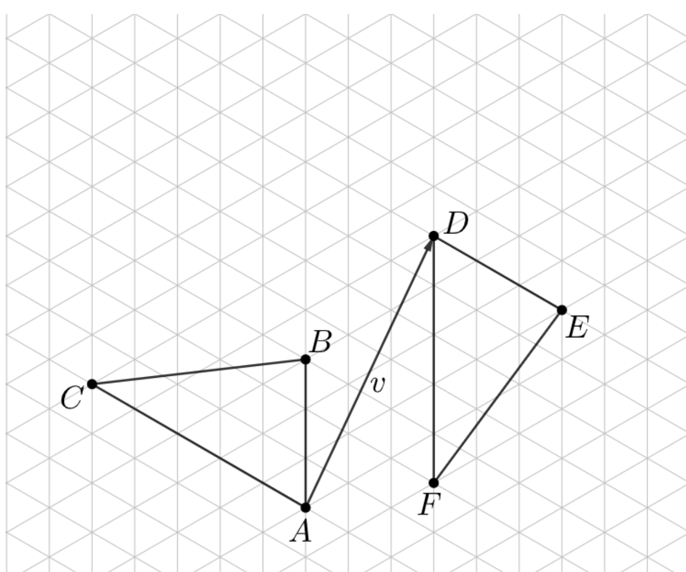
Rotation: a rigid transformation that takes a point to another point on the circle through the original point with a given center by a directed angle

Demonstrate

AJ suspects ΔABC is congruent to ΔDEF. They think these steps will work to show there is a rigid transformation from ABC to DEF:

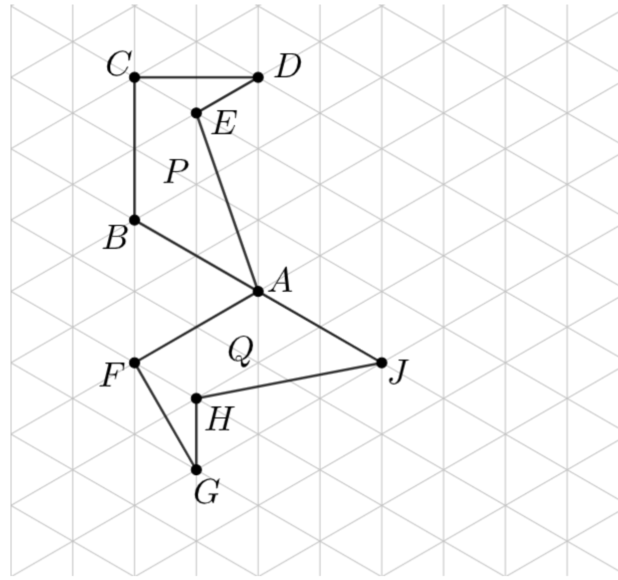
- Translate by directed line segment v
- Rotate the image ____ degrees clockwise around point D
- Reflect that image over segment $\overline{DE}$

Draw each image and determine the angle of rotation needed for these steps to takes ABC to DEF.



Practice

1. Here are 2 polygons, P and Q. Select **all** sequences of translations, rotations, and reflections below that would take polygon P to polygon Q.



A. Rotate 180° around point A.

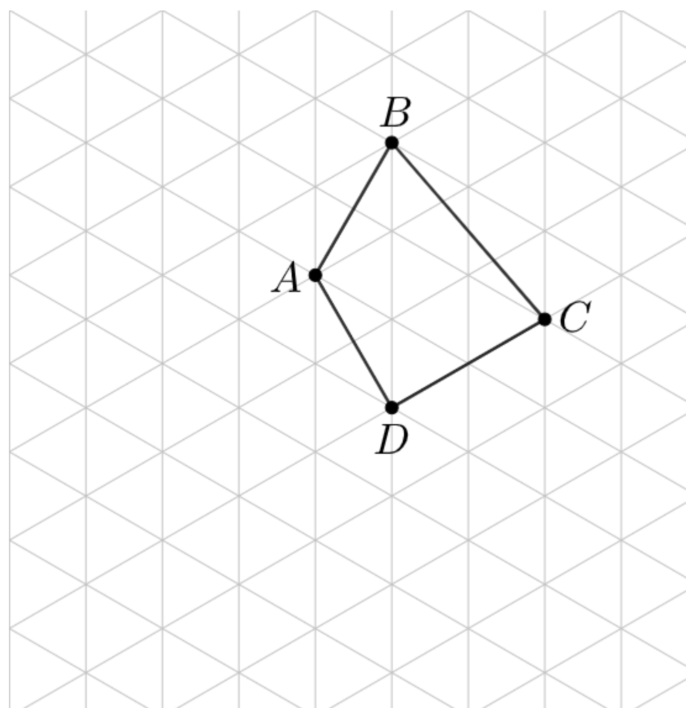
B. Rotate 60° counterclockwise around point A and then reflect over segment $\overline{FA}$.

C. Translate so that A is taken to J. Then reflect over segment $\overline{BA}$.

D. Reflect over segment $\overline{BA}$ and then translate by directed line segment $\overline{BA}$.

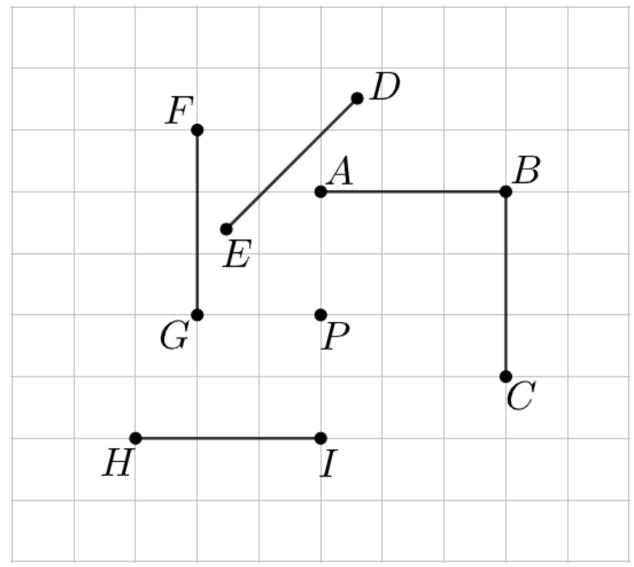
E. Reflect over segment $\overline{BA}$ and then rotate 60° counterclockwise around point A.

2. Draw the image of quadrilateral $ABCD$ when rotated 120° counterclockwise around the point D.



3. There is an equilateral triangle, ΔABC, inscribed in a circle with center D. What is the smallest angle you can rotate ΔABC around D so that the image of A is B?

4. Which segment is the image of $\overline{AB}$ when rotated 90° counterclockwise around point P?



Name: ____________________

Lesson 1.11
Deep Dive: Transformations

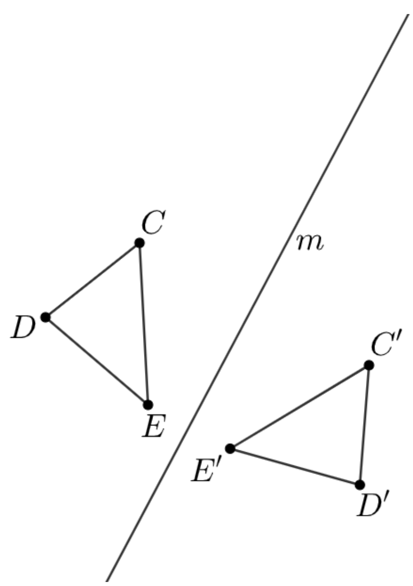
Geometry GT

Task #1

$\Delta C'D'E'$ is the image of ΔCDE after a reflection across line m.

A. Reflect $\Delta C'D'E'$ across line $\overleftrightarrow{CC'}$ and label the new image $\Delta C''D''E''$.

B. Find a single rigid motion that takes ΔCDE to $\Delta C''D''E''$.

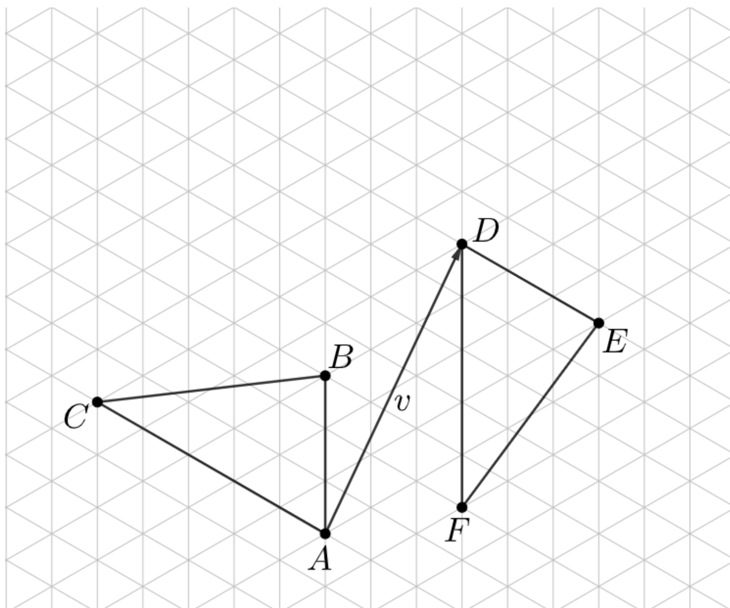


Task #2

AJ suspects ΔABC is congruent to ΔDEF. They think these steps will work to show there is a rigid transformation from ABC to DEF:

- Translate by directed line segment v
- Rotate the image 120° clockwise around point D
- Reflect that image over segment $\overline{DE}$

Draw each image.



A. AJ's first two steps could be combined into a single rotation. What is the center and angle of this rotation?

B. Describe a general procedure for finding a center of rotation.

Areas for Improvement	Standards and Criteria	Areas of Excellence
	Reason abstractly and quantitatively Describe why certain geometric concepts are true and apply the ideas to specific scenarios.	
	Use appropriate tools strategically Accurately use a straightedge, compass, and/or tracing paper to construct the appropriate figures.	
	Attend to precision Use precise mathematical language in writing instructions and utilize accurate definitions.	

Name: ____________________

Lesson 1.12
Symmetry

Geometry GT

Recall

Here is a segment $\overline{AB}$.

If you translate the segment up 5 units then down 5 units, it looks the same as it did originally. What other rigid transformations create an image that fits exactly over the original segment? Are there any *single* rigid motions that do the same thing?

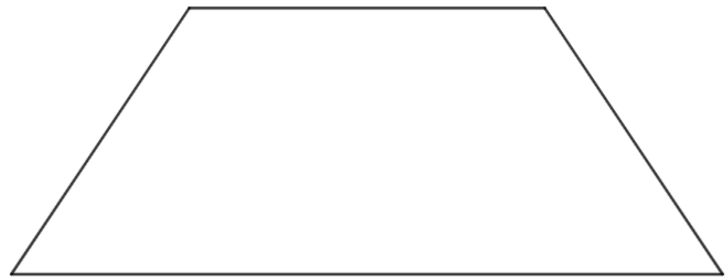
Definitions

Reflectional symmetry: when a figure does not change (the image and the original are the exact same figure) while undergoing a reflection
Line of symmetry: the line that passes through a shape and divides it into congruent halves

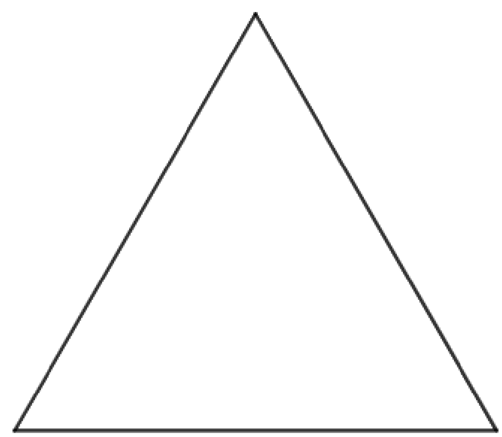
Explore

For each of the following shapes, draw each line of symmetry and describe, in words, each line of symmetry.

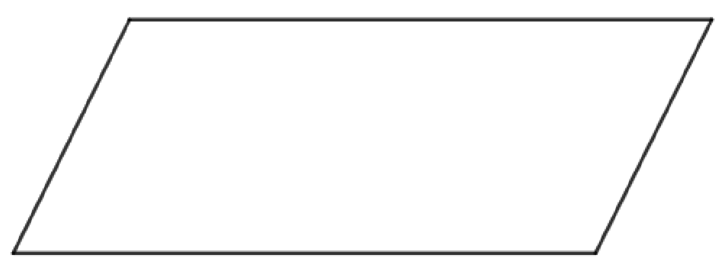
Isosceles trapezoid



Equilateral triangle



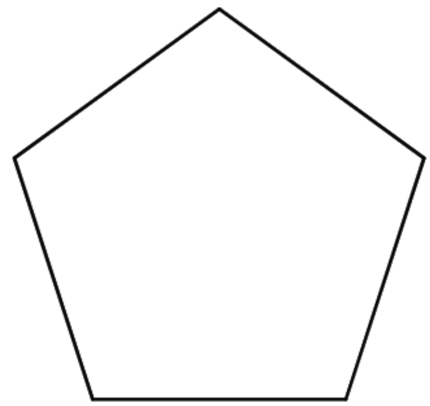
Parallelogram



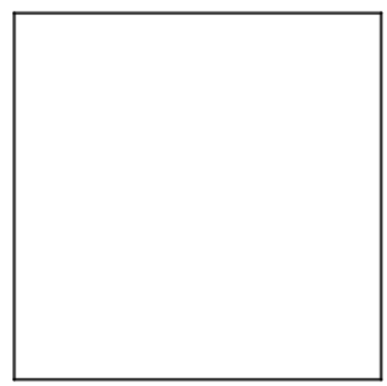
Discuss

For each of the following shapes, draw each rotation that creates symmetry and describe, in words, each rotation, including the center, angle, and direction of rotation.

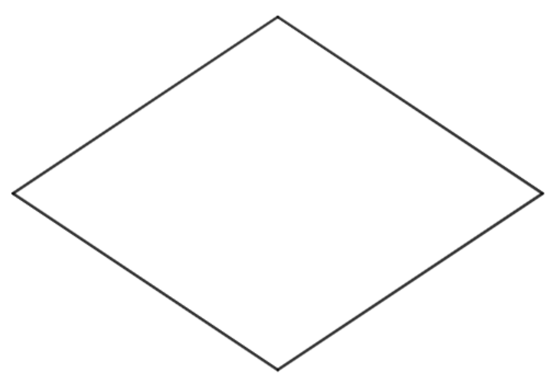
Regular pentagon



Square



Rhombus

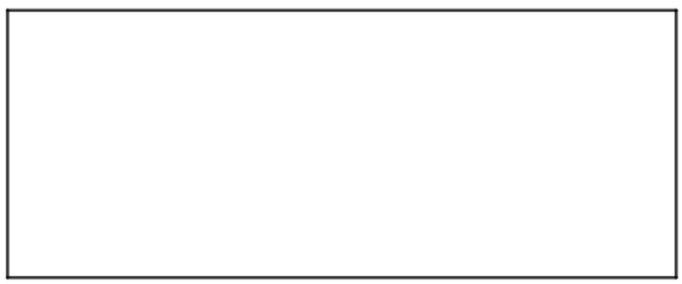


Definition

Rotational symmetry: when a figure does not change (the image and the original are the exact same figure) while undergoing a rotation of less than 360°

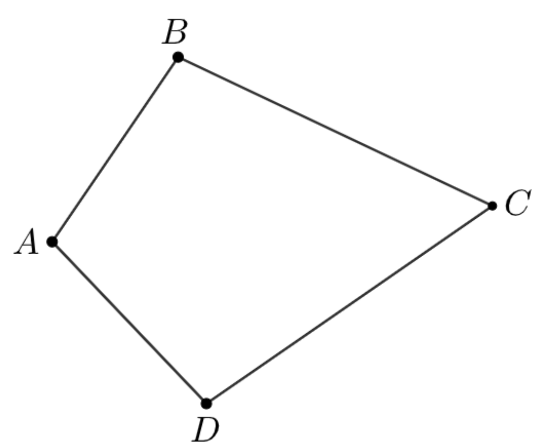
Demonstrate

What happens to the diagonal of a rectangle when the rectangle is reflected across a line of symmetry? What does this suggest about the diagonals of rectangles?



Practice

1. In quadrilateral $BADC$, $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$. The segment $\overline{AC}$ is a line of symmetry for this quadrilateral.



A. Based on the line of symmetry, explain why the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular.

B. Based on the line of symmetry, explain why $\angle ABC$ and $\angle ADC$ have the same measure.

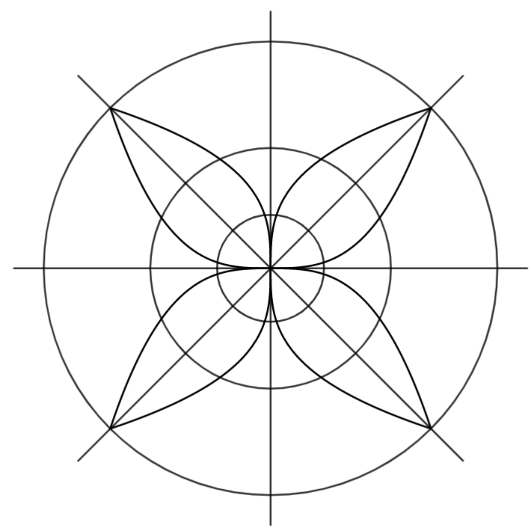
2. A triangle has rotational symmetry that can take any of its vertices to any of its other vertices. Select **all** conclusions that we can reach from this.

A. All sides of the triangle have the same length

B. All angles of the triangle have the same measure

C. All rotations take one half of the triangle to the other half of the triangle

3. Select **all** the angles of rotation that produce symmetry for this graph.



A. 45°

B. 90°

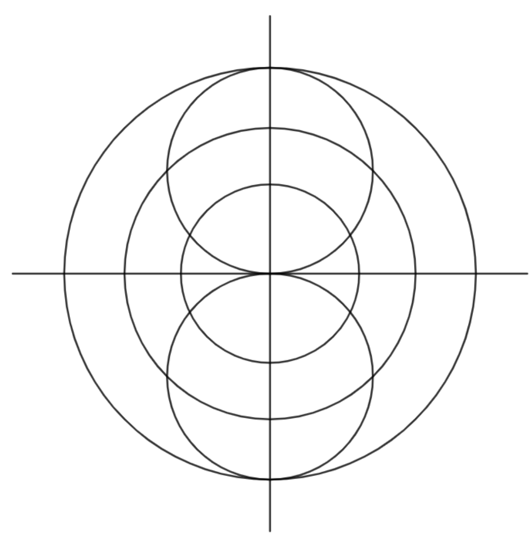
C. 135°

D. 180°

E. 225°

F. 270°

4. Identify any lines of symmetry the figure has.



5. A triangle has a line of symmetry. Select **all** conclusions that *must* be true.

A. All sides of the triangle have the same length

B. All angles of the triangle have the same measure

C. No sides of the triangle have the same length

D. No angles of the triangle have the same measure

E. Two sides of the triangle have the same length

F. Two angles of the triangle have the same measure

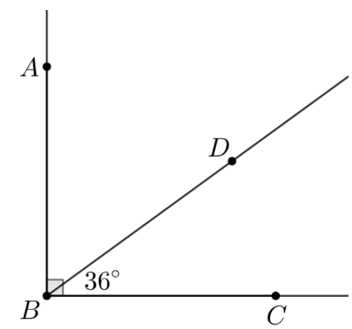
Name: ______________________________

Lesson 1.13
Angle Theorems

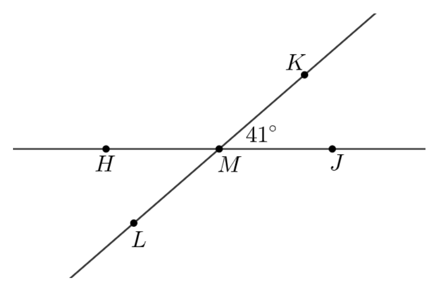
Geometry GT

Recall

In the figure to the right, find $m\angle ABD$.

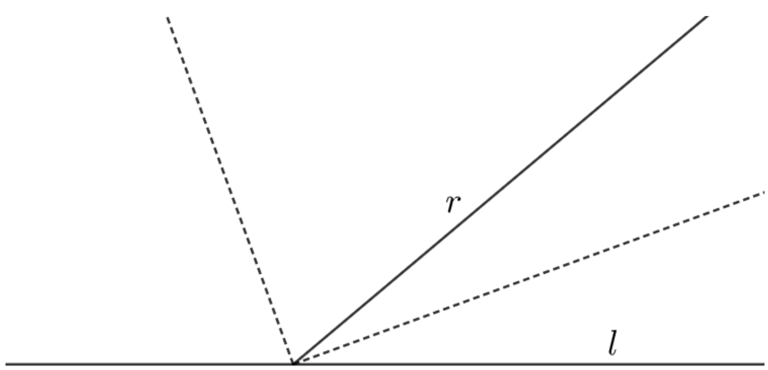


In the figure to the right, find $m\angle HML$ and $m\angle HMK$.



Explore

In the figure below, ray r meets line l, and the dashed lines are angle bisectors.

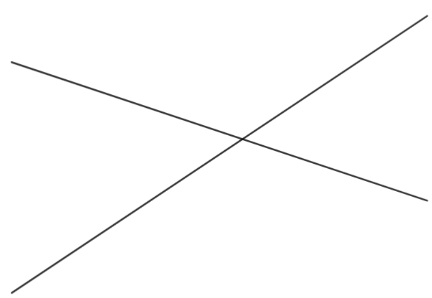


Noel made the conjecture: "The angle formed between the angle bisectors is always a right angle, no matter what the angle between r and l is." It is difficult to tell specifically which angles Noel is talking about in his conjecture. Label the diagram and rephrase Noel's conjecture more precisely using your labels.

Is the conjecture true? Explain your reasoning.

Discuss

Here are two intersecting lines that create two pairs of vertical angles.



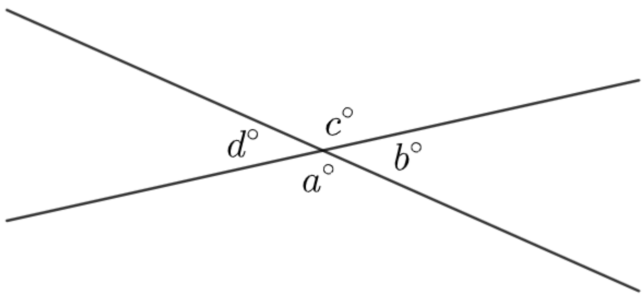
What is the relationship between vertical angles? Write down a conjecture. Label the diagram to make it easier to write your conjecture precisely.

How do you know your conjecture is true for all possible pairs of vertical angles? Explain your reasoning.

Theorem

Vertical angles are congruent.

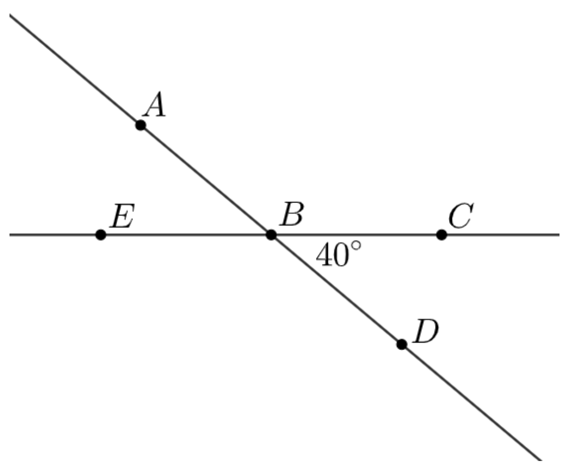
Demonstrate



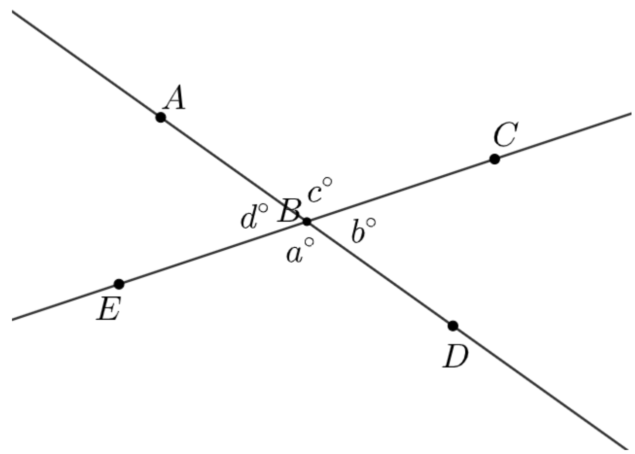
Use algebra to convincingly explain why $a = c$.

Practice

1. What is $m\angle ABE$?



2. Select **all** true statements about the figure.



A. $c + b = d + c$

B. $d + b = 180$

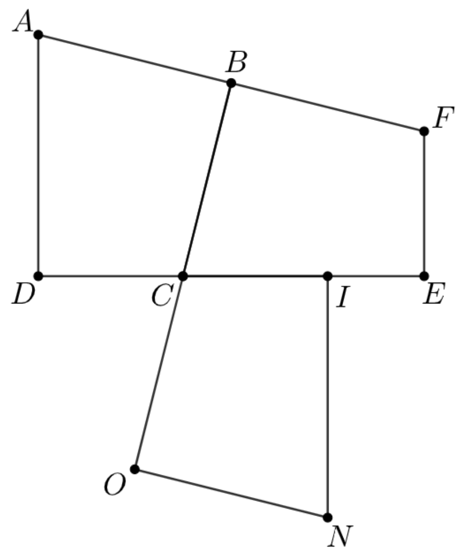
C. Rotate clockwise by $\angle ABC$ using center B, then $\angle CBD$ is the image of $\angle ABE$

D. Rotate 180° clockwise using center B, then $\angle CBD$ is the image of $\angle EBA$

E. Reflect across the angle bisector of $\angle ABC$, then $\angle CBD$ is the image of $\angle ABE$

F. Reflect across line $\overleftrightarrow{CE}$, then $\angle CBD$ is the image of $\angle EBA$

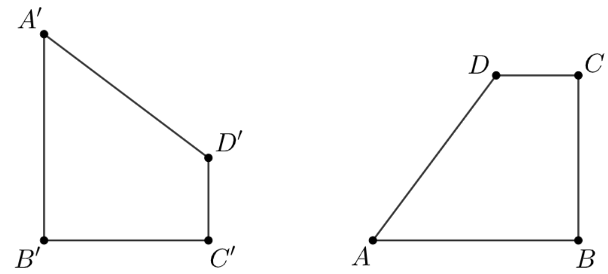
3. Draw the result of this sequence of transformations.



A. Rotate $ABCD$ clockwise by $\angle ADC$ using point D as the center.

B. Translate the image by the directed line segment $\overline{DE}$.

4. Quadrilateral $ABCD$ is congruent to quadrilateral $A'B'C'D'$. Describe a sequence of rigid motions that take A to A', B to B', C to C', and D to D'.



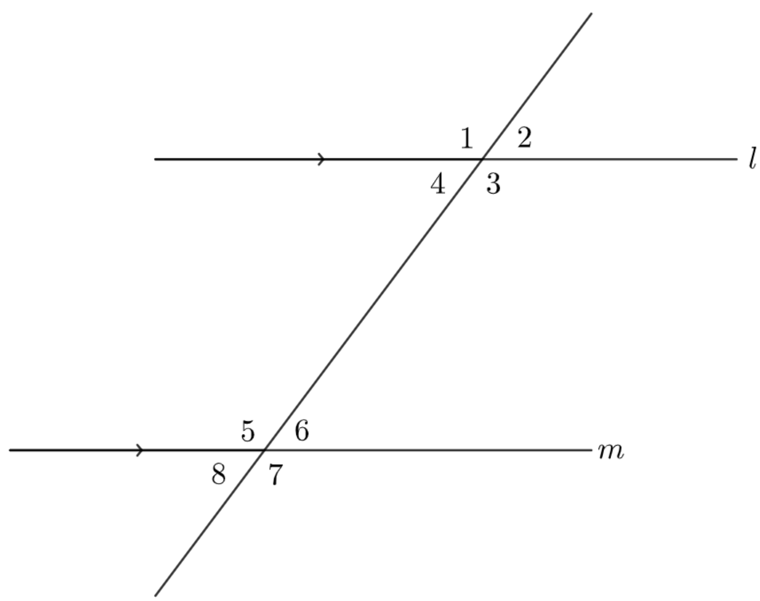
Name: ______________________

Lesson 1.14
Transversals

Geometry GT

Recall

In the figure below, $l \parallel m$.



Identify the relationship between each of the following angle pairs.

A. $\angle 1$ and $\angle 3$

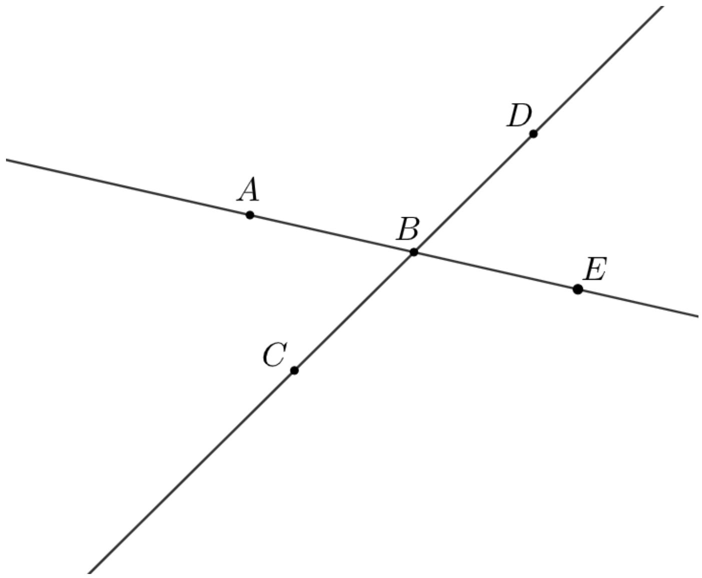
B. $\angle 5$ and $\angle 6$

C. $\angle 4$ and $\angle 8$

D. $\angle 3$ and $\angle 5$

Explore

Lines $\overleftrightarrow{AE}$ and $\overleftrightarrow{CD}$ are intersecting.



Translate lines $\overleftrightarrow{AE}$ and $\overleftrightarrow{CD}$ by the directed line segment from B to C. Label the images of A, B, C, D, and E as A', B', C', D', and E'. What is true about lines $\overleftrightarrow{AE}$ and $\overleftrightarrow{A'E'}$?

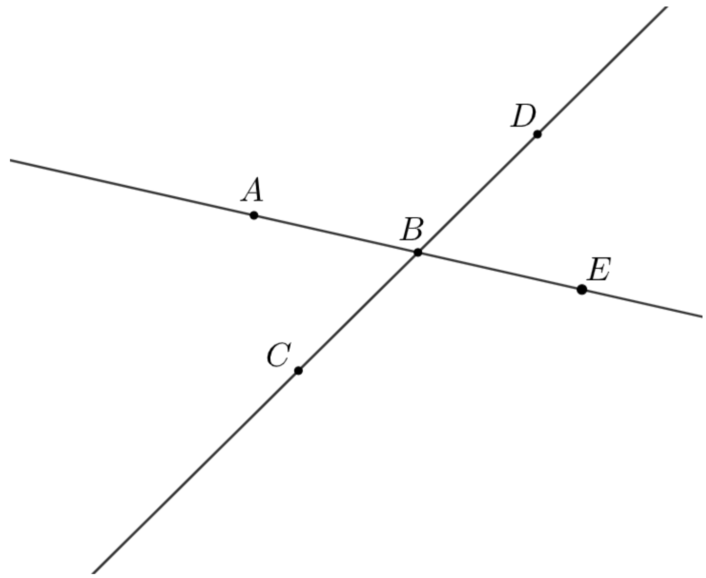
Identify any pairs of congruent angles. Explain your reasoning.

Assertion

Rotations by 180° take lines to parallel lines or themselves.

Discuss

Lines $\overleftrightarrow{AE}$ and $\overleftrightarrow{CD}$ are intersecting.



Rotate line $\overleftrightarrow{AE}$ by 180° around point C. Label the images of A, B, C, D, and E as A', B', C', D', and E'. What is true about lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{A'B'}$?

Identify any pairs of congruent angles. Explain your reasoning.

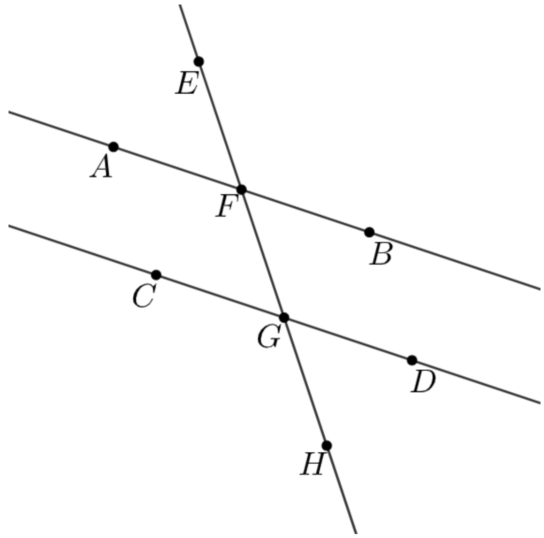
Theorems

Corresponding Angle Theorem: if two parallel lines are cut by a transversal, then corresponding angles are congruent; conversely, if two lines are cut by a transversal and corresponding angles are congruent, then the lines must be parallel

Alternate Interior Angle Theorem: if two parallel lines are cut by a transversal, then alternate interior angles are congruent; conversely, if two lines are cut by a transversal and alternate interior angles are congruent, then the lines must be parallel

Demonstrate

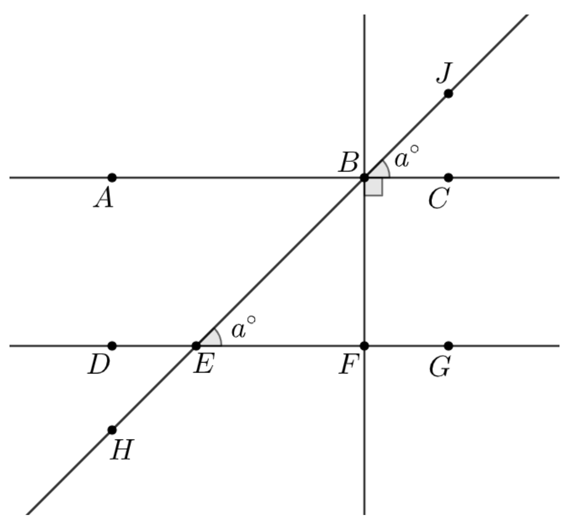
In the figure below, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $m\angle AFE = 14x - 31$, $m\angle CGF = 8x + 5$, and $m\angle CGH = 9y - 35$.



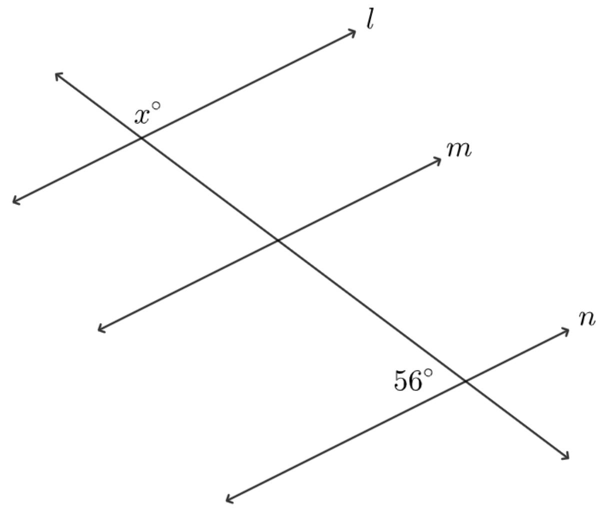
Find the values of x and y.

Practice

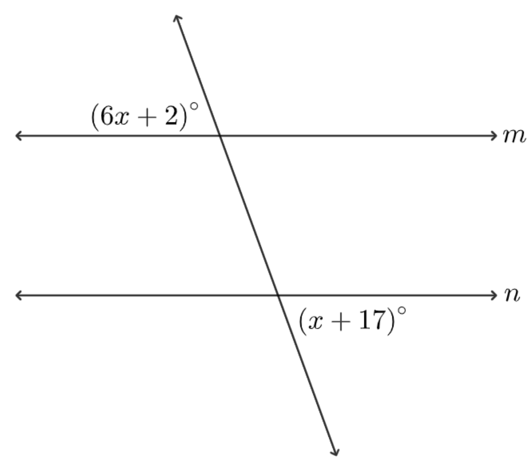
1. Explain why $\overleftrightarrow{AC}$ and $\overleftrightarrow{DG}$ must be parallel.



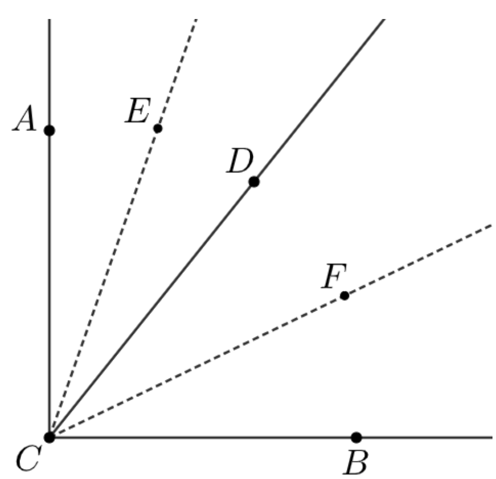
2. Lines l, m, and n are parallel. Find the value of x.



3. Given that $m \parallel n$, find the value of x.



4. Lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$ are perpendicular. The dashed rays bisect angles $\angle BCD$ and $\angle ACD$. Explain why $m\angle ECF = 45°$.



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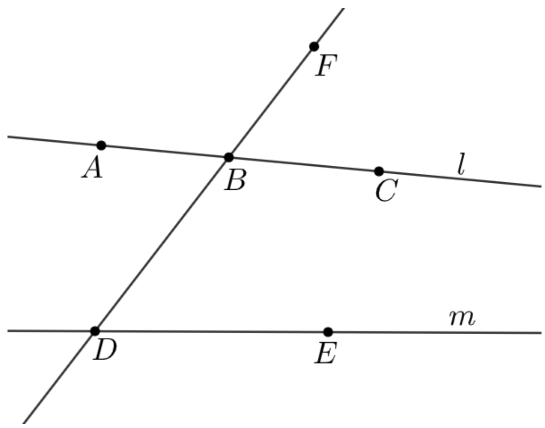
Lesson 1.15
Triangle Theorems

Geometry GT

Recall

In the figure below, lines l and m are *not* parallel and have been cut by a transversal.



Dylan believes $\angle CBF$ is congruent to $\angle BDE$ because they are corresponding angles and a translation along the directed line segment from B to C would take one angle onto the other. Here are his reasons:

- The translation takes B onto D, so the image of B is D
- The translation takes E somewhere on ray $\overrightarrow{DB}$ because it would need to be translated by a distance greater than BD to land on the other side of D
- The image of C has to land somewhere on line m because translations take lines to parallel lines and line m is the only line parallel to l that goes through B'
- The image of C, call it C', has to land on the right side of $\overleftrightarrow{BD}$ or else $\overleftrightarrow{CC'}$ wouldn't be parallel to the directed line segment from B to D

Are the statements true or false? For any false statement, explain why it is not true.

Explore

Use a straightedge to create ΔABC and label the three angle measures as a°, b°, and c°. Use either a straightedge and compass or paper folding to mark the midpoints of two of the sides, and extend the third side in both directions to make a line.

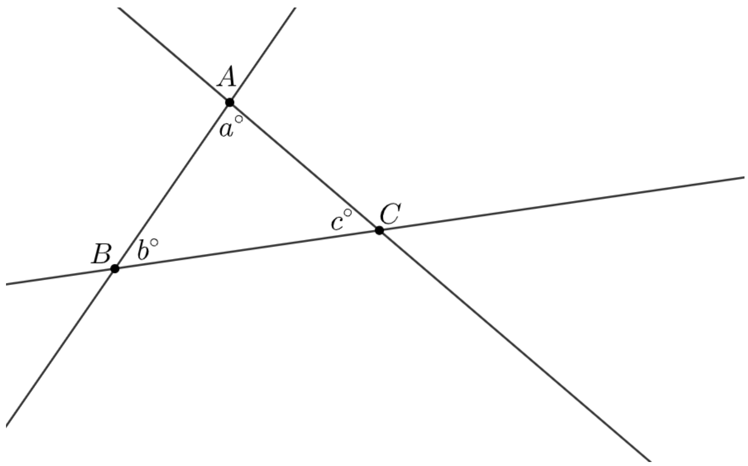
Use what you know about rotations to create a line parallel to the line you made that goes through the opposite vertex. What is the value of $a + b + c$? Explain your reasoning.

Theorem

Triangle Angle Sum Theorem: the three angle measures of any triangle always sum to 180°

Discuss

Here is ΔABC with angle measures $a°$, $b°$, and $c°$. Each side has been extended to a line.



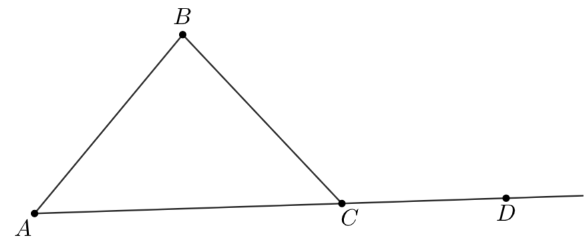
Translate ΔABC along the directed line segment from B to C to make $\Delta A'B'C'$ and label the measures of the angles. Translate $\Delta A'B'C'$ along the directed line segment from A' to C to make $\Delta A''B''C''$ and label the measures of the angles.

Label the measures of the angles that meet at C. Explain your reasoning. What is the value of $a + b + c$?

Demonstrate

Here is ΔABC with point D on $\overrightarrow{AC}$ (but not between A and C).

Explain how you know $m\angle BAC + m\angle ABC = m\angle BCD$.

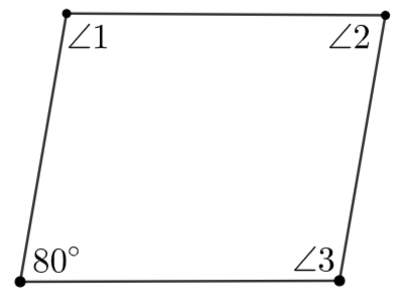


Theorem

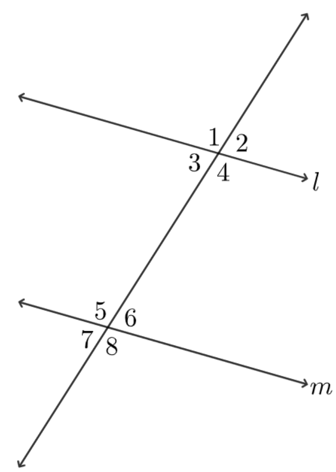
Exterior Angle Theorem: the measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles

Practice

1. The quadrilateral below is a parallelogram. Find $m\angle 1$, $m\angle 2$, $m\angle 3$.



2. In the figure, $l \parallel m$. Identify **all** angles that are congruent to $\angle 2$.



3. In ΔABC, $\overline{AC}$ is extended through C to D. If $m\angle BAC = (8x - 21)^\circ$, $m\angle ABC = (5x + 1)^\circ$, and $m\angle DCB = (7x + 28)^\circ$, what is the value of $m\angle ACB$?