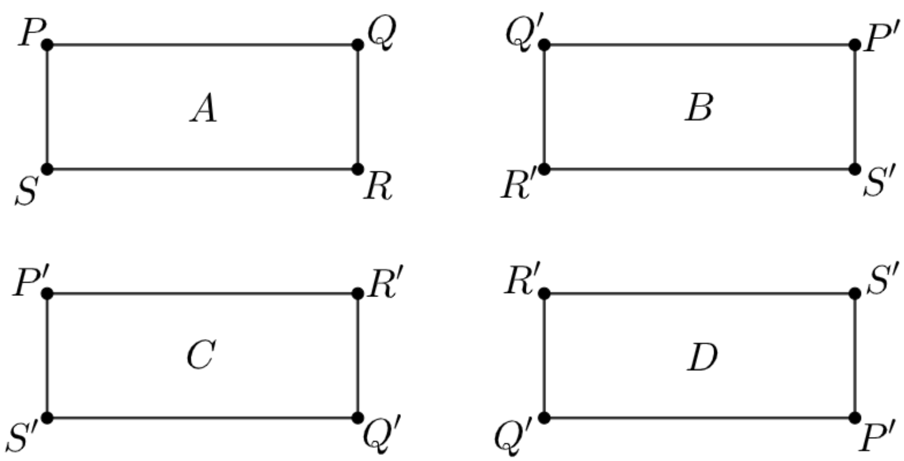
Name: ______________________

Lesson 2.01
Congruent Parts

Geometry GT

Analyze

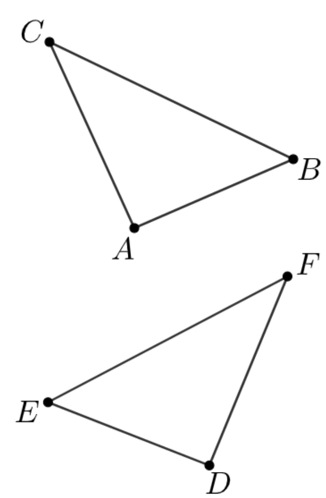
What do you notice? What do you wonder?



Explore

In the figure, ΔABC is congruent to ΔDEF.

A. Find a rigid transformation that takes ΔABC to ΔDEF.

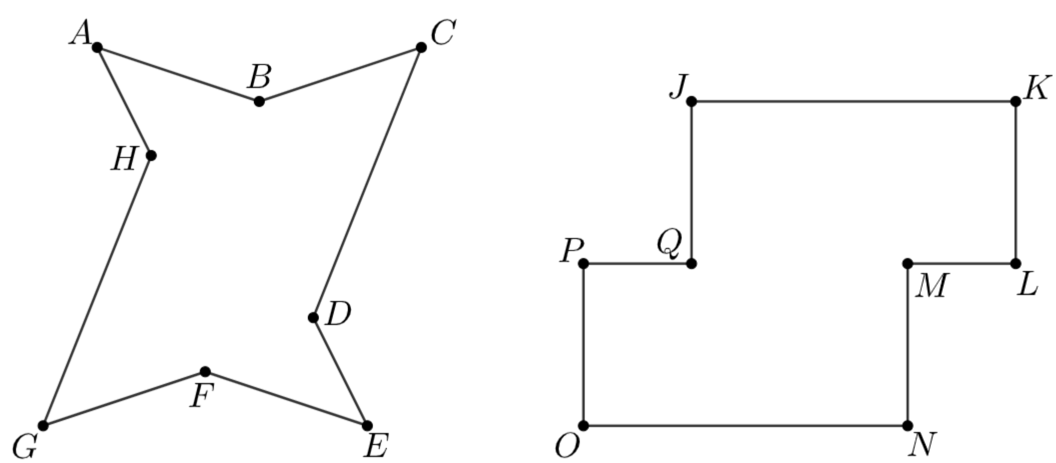


B. What is the image of segment $\overline{BC}$ after that transformation?

C. Explain how you know those segments are congruent.

D. Justify that $\angle ABC \cong \angle DEF$.

For each figure, draw additional line segments to divide the figure into two congruent polygons. Label any new vertices and identify the corresponding vertices of the congruent polygons.

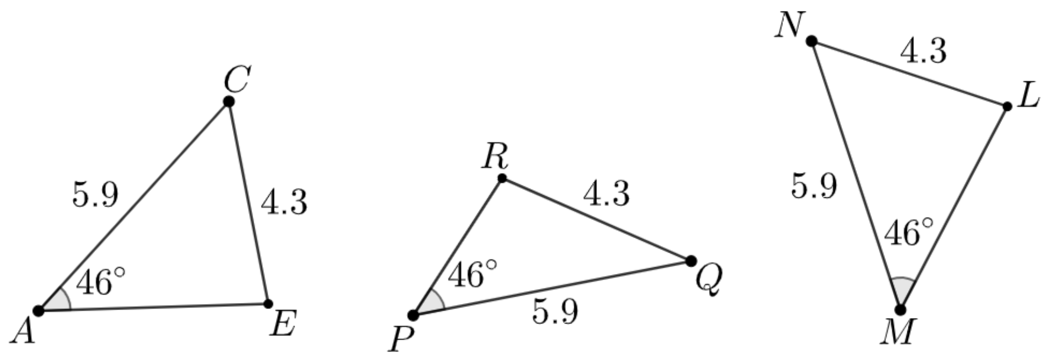


Theorem

If two figures are congruent, then corresponding parts of those figures must be congruent.

Discuss

Here are three triangles.

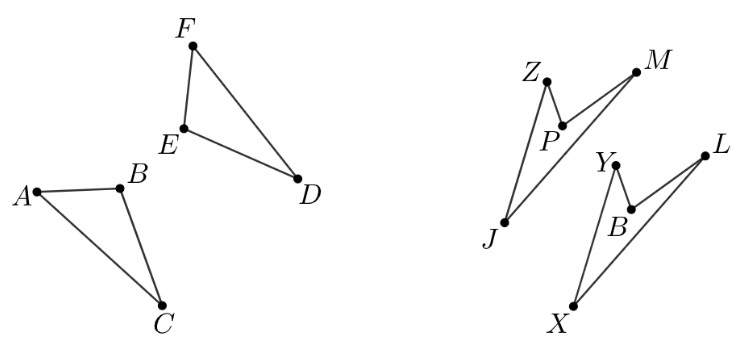


A. ΔACE is congruent to which triangle? Explain your reasoning.

B. Describe a rigid transformation that takes ΔACE to that triangle. Draw each step of the transformation.

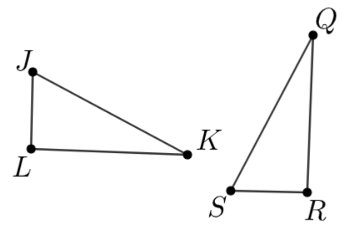
Demonstrate

Each pair of figures is congruent. Decide whether each congruence statement is true or false.

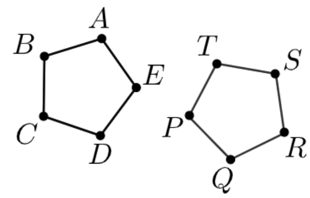


$\Delta ABC \cong \Delta FED$

$PZJM \cong LYXB$



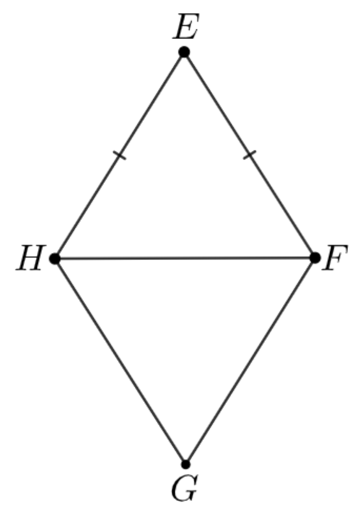
$\Delta JKL \cong \Delta QRS$



$ABCDE \cong PQRST$

Practice

1. Triangle ΔFGH is the image of isosceles triangle ΔFEH after a reflection across segment $\overline{HF}$. Select **all** the statements that are a result of corresponding parts of congruent triangles being congruent.



A. $EFGH$ is a rectangle

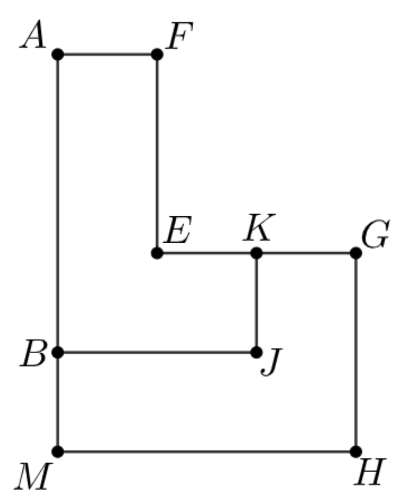
B. $EFGH$ has 4 congruent sides

C. Diagonal $\overline{FH}$ bisects angles $\angle EFG$ and $\angle EHG$

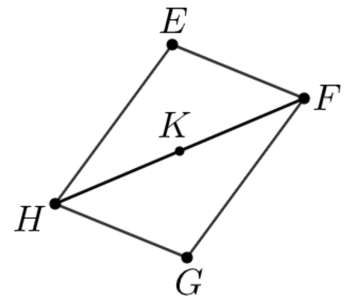
D. Diagonal $\overline{FH}$ is perpendicular to side $\overline{FE}$

E. Angle $\angle FEH$ is congruent to angle $\angle FGH$

2. Figure $MBJKGH$ is the image of figure $AFEKJB$ after being rotated $90°$ counterclockwise about point K. Draw a segment in figure $AFEKJB$ to create a quadrilateral. Draw the image of the segment when rotated $90°$ counterclockwise about point K. Write a congruence statement for the quadrilateral you created in figure $AFEKJB$ and the image of the quadrilateral in figure $MBJKGH$.



3. Triangle ΔHEF is the image of triangle ΔFGH after a 180° rotation about point K. Select **all** statements that must be true.



A. $\Delta FGH \cong \Delta FEH$

B. $\Delta EFH \cong \Delta GFH$

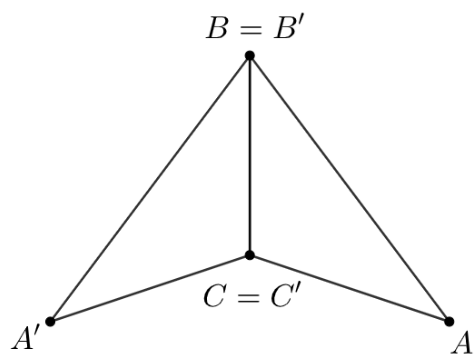
C. $\angle KHE \cong \angle KFG$

D. $\angle GHK \cong \angle KHE$

E. $\overline{EH} \cong \overline{FG}$

F. $\overline{GH} \cong \overline{EF}$

4. Triangle $\Delta A'B'C'$ is a reflection of triangle ΔABC across line $\overleftrightarrow{BC}$. Justify why $\overrightarrow{BC}$ is the angle bisector of angle $\angle ABA'$.



Name: ____________________

Lesson 2.02
Congruent Triangles

Geometry GT

Recall

If triangle ΔABC is congruent to $\Delta A'B'C'$...

A. What must be true?

B. What could possibly be true?

C. What definitely can't be true?

Explore

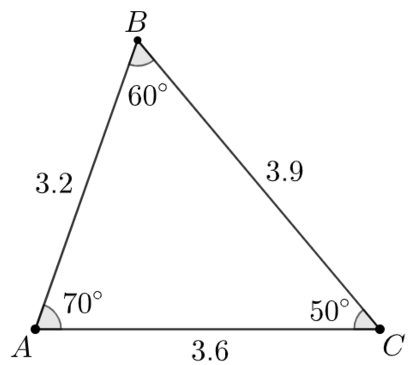
Draw ΔABC with the following measurements:

- $m\angle A = 40°$
- $m\angle B = 20°$
- $m\angle C = 120°$
- $AB = 5\text{cm}$
- $AC = 2\text{cm}$
- $BC = 3.7\text{cm}$

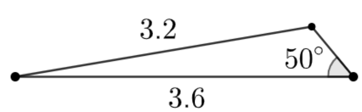
Highlight each piece of given information that you used. Check your triangle to make sure the remaining measurements match.

Discuss

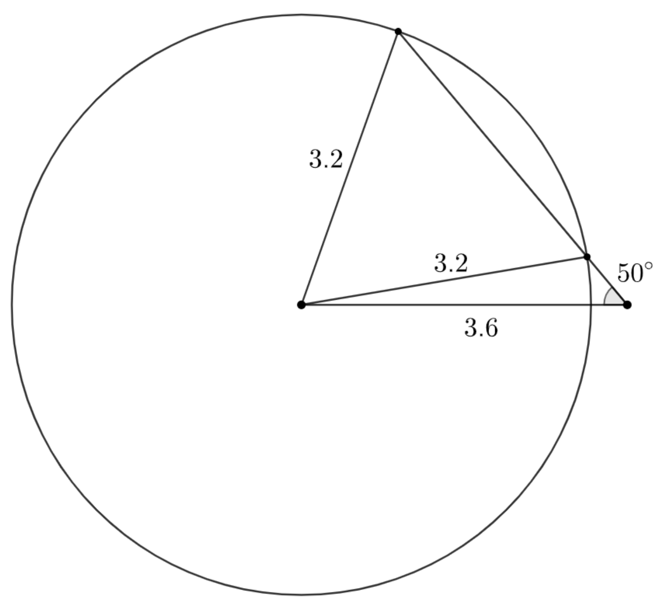
Kenan was attempting to draw a triangle that Josh drew without looking at it.



Kenan asks, "can I have two sides and an angle?" Josh said the sides were 3.2cm and 3.6cm, and the angle was 50°.



Is Kenan's triangle congruent to Josh's triangle? Did Kenan do anything that didn't match Josh's description? How could Kenan have been more specific in his request?



Demonstrate

Draw a triangle with the following measurements:

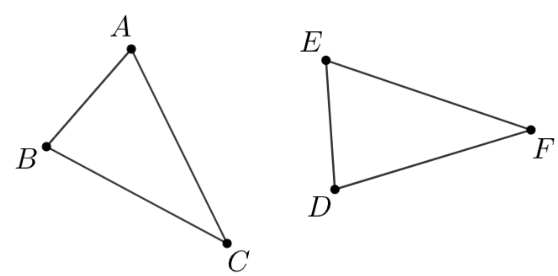
- $m\angle A = 143°$
- $m\angle B = 16°$
- $m\angle C = 21°$

Compare your triangle with a neighbor's. Are they congruent?

What additional information might you need?

Practice

1. Triangle ΔABC is congruent to triangle ΔEDF. Thus, there is a sequence of rigid motions that takes ΔABC to ΔEDF. Select **all** true statements after the transformation.



A. $\angle A$ coincides with $\angle F$

B. $\angle B$ coincides with $\angle D$

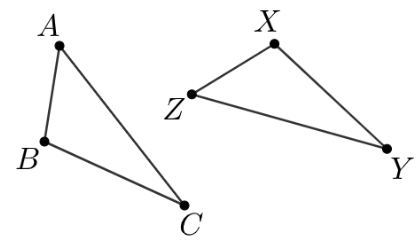
C. $\overline{AC}$ coincides with $\overline{EF}$

D. $\overline{BC}$ coincides with $\overline{ED}$

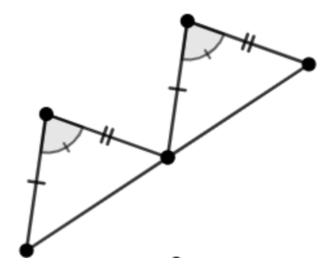
E. $\overline{AB}$ coincides with $\overline{ED}$

2. Sketch the unique triangles that can be made with angles measuring $40°$ and $100°$ and side length 3. How do you know you have sketched all possibilities?

3. In the figure, $\Delta ABC \cong \Delta ZXY$. Describe a sequence of rigid motions that will take ΔABC onto ΔZXY.

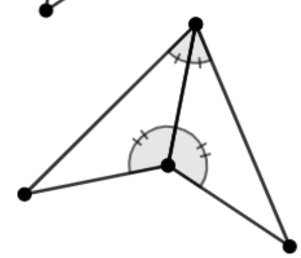


4. Match each statement using only the information shown in the pairs of congruent triangles.

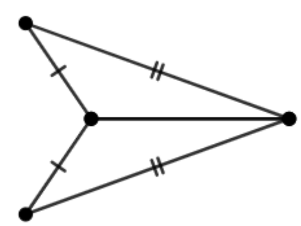


A. In the two triangles there are three pairs of congruent sides

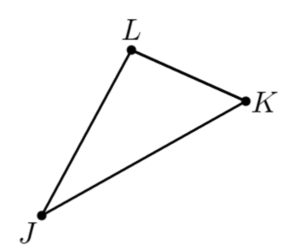
B. The two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle



C. The two angles and the included side of one triangle are congruent to two angles and the included side of another triangle



5. What is the least amount of information that you need to construct a triangle congruent to this one?



Name: ______________________

Lesson 2.03
Deep Dive: Evidence for Proofs

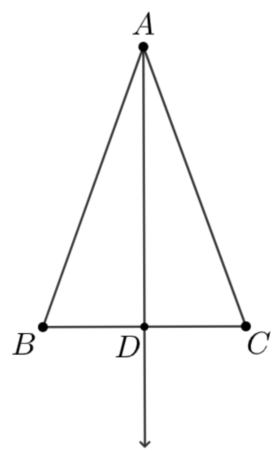
Geometry GT

Theorem

If two segments have the same length, then they are congruent.

Task #1

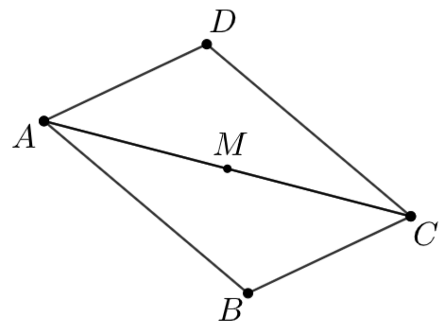
In a triangle, a ray from a vertex is both an angle bisector and a perpendicular bisector.



In the figure, what information must be true? Include an explanation with each piece of information.

Task #2

A triangle is rotated 180° around the midpoint of one of the sides.



In the figure, what information must be true? Include an explanation with each piece of information.

Name: ____________________

Lesson 2.04
SAS Congruence

Geometry GT

Recall

A triangle has an angle measuring 35° and a side adjacent to the angle with a length of 3cm. Can you create two triangles with those measurements that are **not** congruent?

Explore

Two triangles have two pairs of corresponding sides congruent, and the corresponding angles between those sides (known as an included angle) are congruent. Sketch two triangles, ΔLMN and ΔPQR, such that $\overline{LM} \cong \overline{PQ}$, $\overline{LN} \cong \overline{PR}$, and $\angle L \cong \angle P$.

Use a sequence of rigid motions to take ΔLMN onto ΔPQR. Draw each image in a different color.

Try to draw a third triangle, ΔXYZ, that is **not** congruent to ΔLMN and ΔPQR.

Theorem

Side-Angle-Side Triangle Congruence Theorem: in two triangles, if two pairs of corresponding sides are congruent, and the corresponding pair of included angles are congruent, then the two triangles are congruent

Definitions

Isosceles triangle: a triangle with two congruent sides
Auxiliary line: an additional line drawn to reveal information

Discuss

Prove that if a triangle is isosceles, then the base angles are congruent. Starting with an isosceles triangle, draw an auxiliary line to create two triangles and use the SAS Triangle Congruence Theorem.

Theorem

Isosceles Triangle Theorem: in an isosceles triangle, the angles opposite the congruent sides are congruent

Demonstrate

It follows from the Side-Angle-Side Triangle Congruence Theorem that if the length of two sides of a triangle are known, and the measure of the included angle is known, then there can only be one possible length for the third side.

Suppose a triangle has sides of lengths 5cm and 12cm. What is the longest the third side could be? What is the shortest it could be?

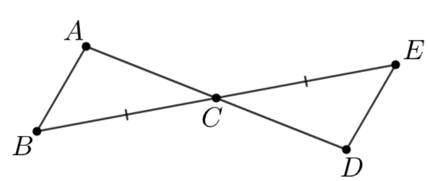
Practice

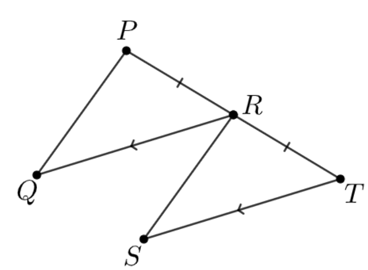
1. Malachi is attempting to prove that quadrilateral $ABCD$ is a parallelogram. He knows that $\overline{AB} \cong \overline{DC}$ and $\angle ABC \cong \angle ADC$. Can he use the Side-Angle-Side Triangle Congruence Theorem to say that $\Delta ABC \cong \Delta ADC$ since $\overline{AC} \cong \overline{AC}$? Why or why not?

2. Conjecture: if a point is on the perpendicular bisector of a segment, then that point must be equidistant from the endpoints of the segment.

> **A.** Sketch and label a diagram of the situation. Mark any information you know, such as segments and angles being congruent.
>
> **B.** Find (or add auxiliary lines to find) two triangles that appear congruent. Shade them in using different colors.
>
> **C.** Do you have enough information to prove that the two triangles are congruent? Explain.

3. For each of the figures below, identify what additional information you would need to use the Side-Angle-Side Triangle Congruence Theorem.





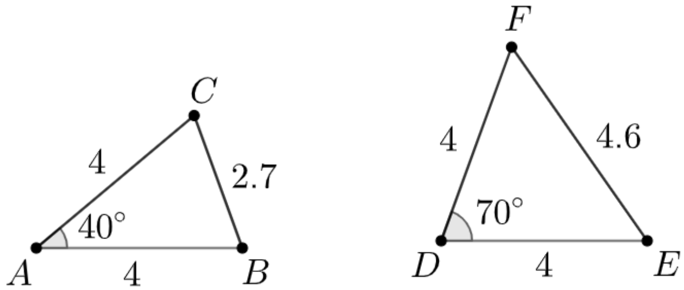
Name: ______________________

Lesson 2.05
AAS/ASA Congruence

Geometry GT

Analyze

What do you notice about the angles and sides in the two triangles below? What do you wonder?



Theorem

In a triangle, the greater side is opposite the greater angle. Conversely, the greater angle is opposite the greater side.

Explore

Triangle ΔABC has the following characteristics:

- $m\angle A = 30°$
- $m\angle B = 95°$
- $AB = 90$ mm
- $BC = 55$ mm
- $AC = 109.5$ mm

Construct triangle ΔABC with dry pasta. Then, attempt to construct a triangle with a longer $\overline{AC}$, and another triangle with a shorter $\overline{AC}$. What happens to the angles and sides of the triangles?

Theorem

Angle-Angle-Side Triangle Congruence Theorem: in two triangles, if two pairs of corresponding angles are congruent, and a corresponding pair of non-included sides are congruent, then the two triangles are congruent

Discuss

Two triangles, ΔWXY and ΔDEF, have two pairs of corresponding angles congruent ($\angle W \cong \angle D$ and $\angle X \cong \angle E$), and the corresponding sides between those angles are congruent ($\overline{WX} \cong \overline{DE}$).
Sketch these two triangles, and use a sequence of rigid motions to take ΔWXY to ΔDEF. Consider how you know that the vertices must line up.

Theorem

Angle-Side-Angle Triangle Congruence Theorem: in two triangles, if two pairs of corresponding angles are congruent, and the corresponding pair of included sides are congruent, then the two triangles are congruent

Demonstrate

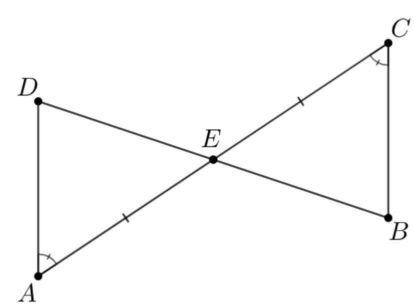
Prove that if a point C is the same distance from A as it is from B, then C must be on the perpendicular bisector of $\overline{AB}$. Hint: sketch the scenario, then consider what auxiliary lines will assist.

Theorem

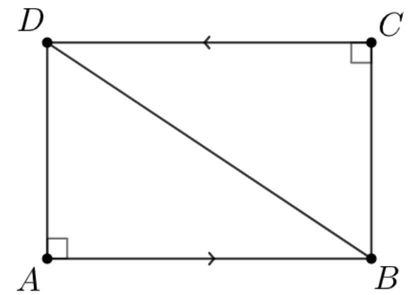
Perpendicular Bisector Theorem: if a point is equidistant from the endpoints of a segment, then it must be on the perpendicular bisector of the segment

Practice

1. What triangle congruence theorem could you use to prove $\Delta ADE \cong \Delta CBE$?



2. Esther wrote a proof that $\Delta BCD \cong \Delta DAB$, but it is incomplete. How can Esther fix her proof?



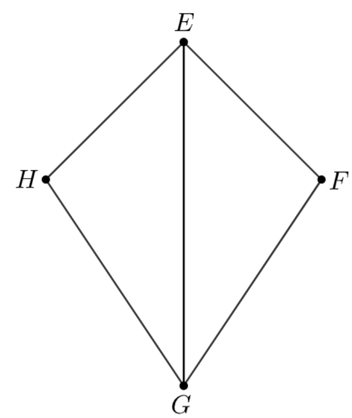
A. Line $\overleftrightarrow{AB}$ is parallel to line $\overleftrightarrow{DC}$ and cut by transversal $\overline{BD}$. So angles $\angle CDB$ and $\angle ABD$ are alternate interior angles and must be congruent.

B. Side $\overline{DB}$ is congruent to side $\overline{BD}$ because they're the same segment.

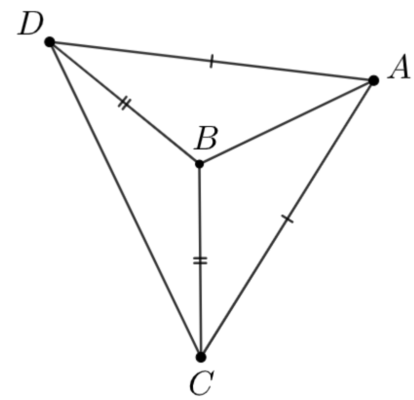
C. $\angle A$ is congruent to $\angle C$ because they're both right angles.

D. By the Angle-Side-Angle Triangle Congruence Theorem, ΔBCD is congruent to ΔDAB.

3. Segment $\overline{GE}$ is an angle bisector of both $\angle HEF$ and $\angle FGH$. Prove that ΔHGE is congruent to ΔFGE.



4. Triangles ΔACD and ΔBCD are isosceles. If $m\angle BAC = 33°$ and $m\angle BDC = 35°$, find $m\angle ABD$.



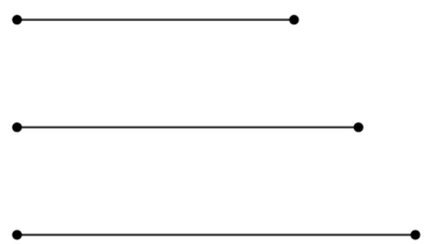
Name: ______________________

Lesson 2.06
SSS Congruence

Geometry GT

Experiment

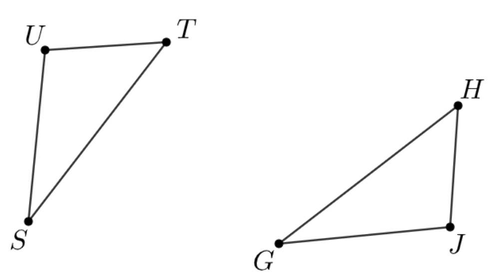
Construct a triangle with the given side lengths on patty paper.



Can you make one that doesn't look like anyone else's?

Explore

Claire is attempting to prove that there is a sequence of rigid motions that take ΔSTU to ΔGHJ, given that $\overline{ST} \cong \overline{GH}$, $\overline{TU} \cong \overline{HJ}$, and $\overline{SU} \cong \overline{GJ}$.



Help fill in the missing pieces to Claire's proof.

A. $\overline{ST}$ is the same length as _______ , so they are congruent. Therefore, there is a rigid motion that takes $\overline{ST}$ to _______ .

B. Apply this rigid motion to ΔSTU. The image of T will coincide with _____ , and the image of S will coincide with _____ .

C. We cannot be certain that the image of U, which we will call U', coincides with _____ yet. If it does, then our rigid motion takes ΔSTU to ΔGHJ, proving $\Delta STU \cong \Delta GHJ$. If it does not, then we continue.

D. $\overline{HJ}$ is congruent to the image of _______ , because rigid motions preserve distance.

E. Therefore, H is equidistant from U' and _____ .

F. A similar argument shows that G is equidistant from U' and _____ .

G. $\overline{GH}$ is the ________________________________ of segment $\overline{U'J}$, because the ________________________________ is determined by two points that are both equidistant from the endpoints of a segment.

H. Reflecting across the ________________________________ of $\overline{U'J}$ takes _____ to _____ .

I. Therefore, after the reflection, all three pairs of vertices coincide, proving triangles __________ and __________ are congruent.

Theorem

Side-Side-Side Triangle Congruence Theorem: in two triangles, if all three pairs of corresponding sides are congruent, then the two triangles are congruent

Discuss

It follows from the Side-Side-Side Triangle Congruence Theorem that, if the lengths of three sides of a triangle are known, then the measures of all the angles must be determined.
On a separate sheet of paper, use a ruler and protractor to make triangles where two sides are 4 cm and the third side is the length given in the table below, then measure the angle between the 4 cm sides.

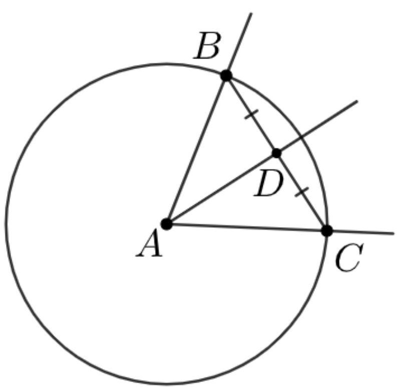
Side Length	Angle Measure	Side Length	Angle Measure
1 cm		5 cm	
2 cm		6 cm	
3 cm		7 cm	
4 cm			

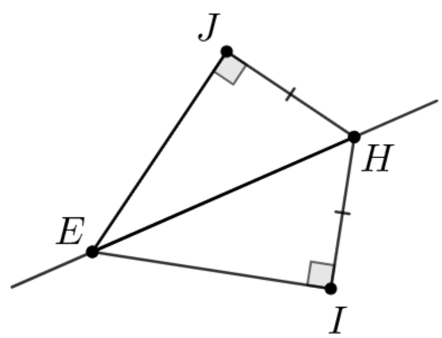
Do you notice any relationships between the side lengths and angle measures?

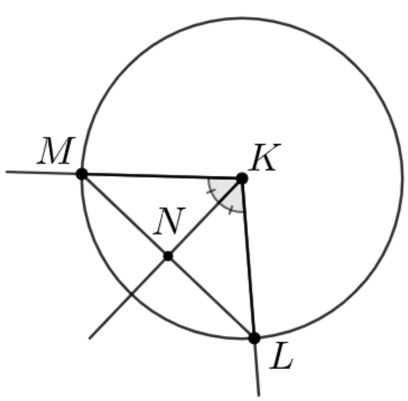
Demonstrate

Label each of the following by whether you could prove the triangles congruent using:

- Side-Side-Side Triangle Congruence Theorem
- Side-Angle-Side Triangle Congruence Theorem
- Angle-Side-Angle Triangle Congruence Theorem
- Angle-Angle-Side Triangle Congruence Theorem
- None of the above

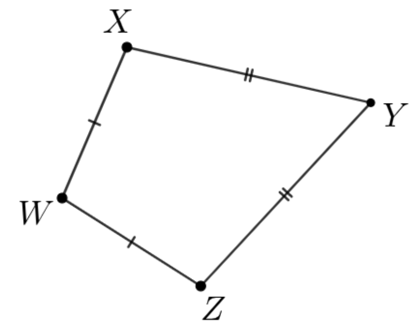




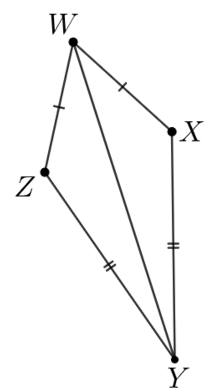


Practice

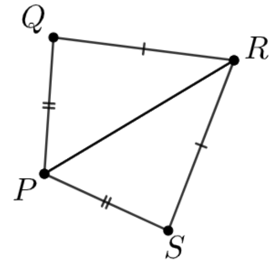
1. A kite is a quadrilateral which has two adjacent sides that are congruent and the other two adjacent sides are also congruent. Given kite $WXYZ$, show that at least one of the diagonals of a kite decomposes the kite into two congruent triangles.



2. $WXYZ$ is a kite. Given $m\angle WXY = 133°$ and $m\angle ZWX = 60°$, find $m\angle ZYW$.



3. Lorin has proven that ΔPRS is congruent to ΔPRQ using the Side-Side-Side Triangle Congruence Theorem. Why can she now conclude that diagonal $\overline{PR}$ bisects angles $\angle SPQ$ and $\angle SRQ$?



4. Each statement is always true. Select **all** statements for which the converse is also always true.

A. Statement: if two angles form a straight angle, then they are supplementary.
Converse: if two angles are supplementary, then they form a straight angle.

B. Statement: in an isosceles triangle, the base angles are congruent.
Converse: if the base angles of a triangle are congruent, then the triangle is isosceles.

C. Statement: if a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.
Converse: if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

D. Statement: if two angles are vertical, then they are congruent.
Converse: if two angles are congruent, then they are vertical.

E. Statement: if two lines are perpendicular, then they intersect to form four right angles.
Converse: if two lines intersect to form four right angles, then they are perpendicular.

Name: ______________________

Lesson 2.07
Deep Dive: Paragraph Proofs

Geometry GT

Definition

Parallelogram: a quadrilateral with two pairs of opposite sides that are parallel

Task #1

Prove that the opposite sides in a parallelogram are congruent.

Task #2

Prove that a quadrilateral with perpendicular diagonals that bisect each other is equilateral.

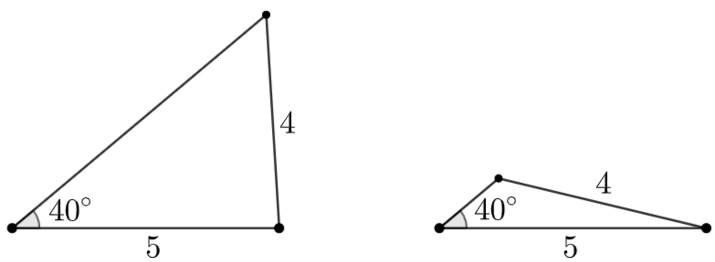
Name: ______________________

Lesson 2.08
SSA/HL Congruence

Geometry GT

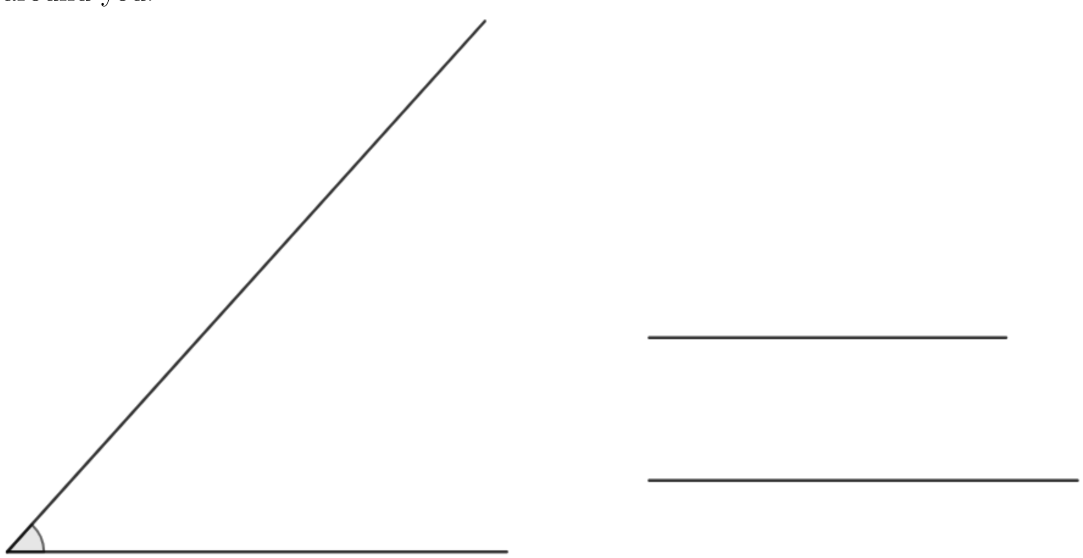
Analyze

What do you notice? What do you wonder?



Explore

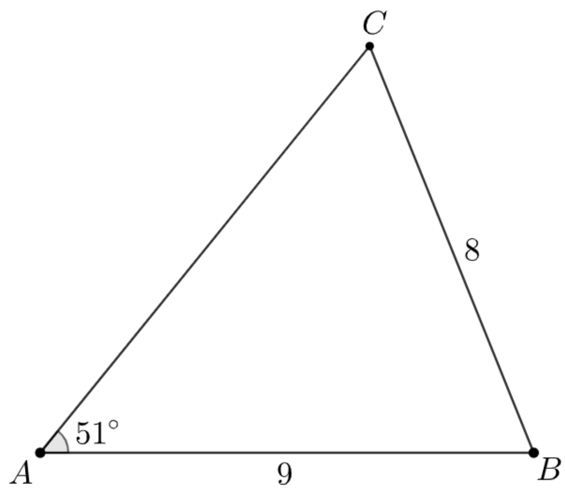
Copy the two segments below and use them to create a triangle using the angle below, and make sure the angle is *not* between the two given sides. Draw the triangle on patty paper, and try to make yours different from those around you.



Is knowing any two sides and an angle enough to guarantee that copies of a triangle will be congruent?

Discuss

Triangle ΔABC is shown below. Use a straightedge and compass to construct point D on $\overleftrightarrow{AC}$ such that $\overline{BD}$ has the same length as $\overline{BC}$.



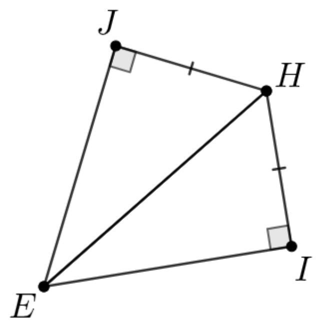
Now use the straightedge and compass to construct the midpoint of $\overline{CD}$, and label it M. Explain why ΔABM must be a right triangle.

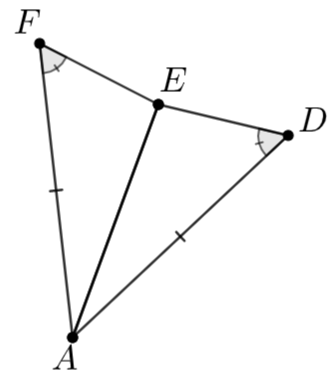
Theorem

Hypotenuse-Leg Triangle Congruence Theorem: in two right triangles, if two pairs of corresponding sides are congruent, and one of the pairs are the sides opposite the right angles, then the two triangles are congruent

Demonstrate

Determine if the following pairs of triangles must be congruent or if they might be congruent.





Practice

1. Which of the following criteria *always* proves triangles congruent? Select **all** that apply.

A. Three pairs of congruent angles

B. Three pairs of congruent sides

C. Two pairs of congruent sides and the pair of included angles

D. Two pairs of congruent sides and a pair of non-included angles

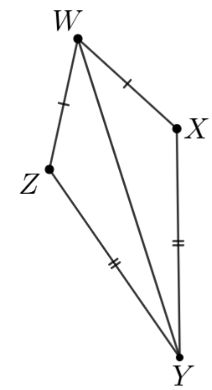
E. Two pairs of congruent angles and the pair of included sides

2. Here are some measurements for ΔABC and ΔXYZ:

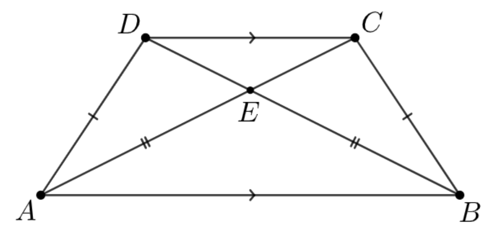
- $m\angle ABC = m\angle XYZ = 30°$
- $BC = YZ = 6$ units
- $CA = ZX = 4$ units

Construct two triangles with the given measurements that are *not* congruent.

3. Emma states that diagonal $\overline{WY}$ bisects $\angle ZWX$ and $\angle ZYX$. Is she correct? Explain your reasoning.



4. Select **all** true statements based on the diagram.



A. $\angle CBE \cong \angle DAE$

B. $\angle CEB \cong \angle DEA$

C. $\overline{DA} \cong \overline{CB}$

D. $\overline{DC} \cong \overline{AB}$

E. $\overleftrightarrow{DC} \parallel \overleftrightarrow{AB}$

F. $\overleftrightarrow{DA} \parallel \overleftrightarrow{CB}$

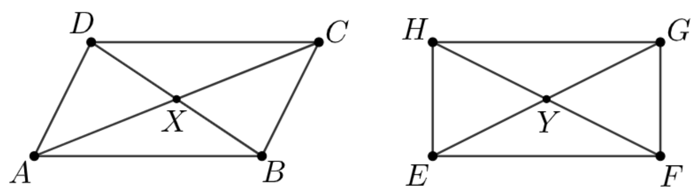
Name: ______________________

Lesson 2.09
Quadrilaterals & Parallelograms

Geometry GT

Analyze

Here is parallelogram $ABCD$ and rectangle $EFGH$. What do you notice? What do you wonder?



Definitions

Rectangle: a quadrilateral with four right angles
Rhombus: a quadrilateral with four congruent sides

Theorem

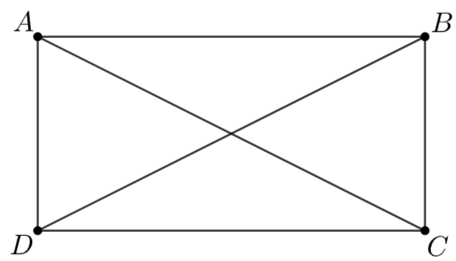
All rectangles are parallelograms

Explore

Conjecture: if a parallelogram has a right angle, then it must be a rectangle. Draw a diagram, and explain why it is true.

Discuss

Conjecture: if the diagonals of a parallelogram are congruent, then it must be a rectangle.



With a partner, work backwards from the conjecture until you are confident that you can prove it is a rectangle using only the given information. Start with the sentence: "I would know $ABCD$ is a rectangle if I knew ______ ." Then continue with the sentence: "I would know [previous statement] if I knew ______ ."

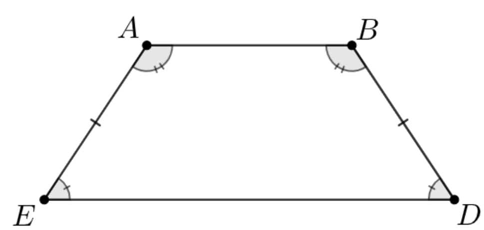
Write down each statement below. If you get stuck, go back one statement and try a different path forwards.

Demonstrate

Write a proof for the previous conjecture.

Practice

1. $ABDE$ is an isosceles trapezoid. Select **all** pairs of congruent triangles.



A. ΔABE and ΔDBE

B. ΔABD and ΔDAE

C. ΔABE and ΔBAD

D. ΔAED and ΔBDE

E. ΔEAB and ΔEDB

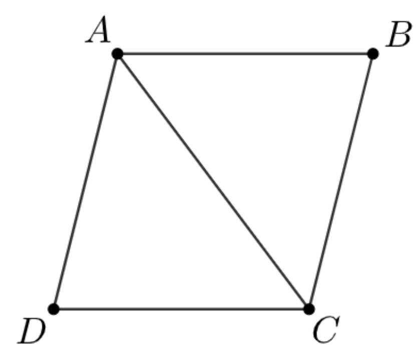
2. Conjecture: a quadrilateral with one pair of sides both congruent and parallel is a parallelogram.

A. Draw a diagram of the situation.

B. Mark the given information.

C. Restate the conjecture as a specific statement using the diagram.

3. In quadrilateral $ABCD$, $\overline{AD}$ is both congruent and parallel to $\overline{BC}$. Show that $ABCD$ is a parallelogram.



Name: ______________________

Lesson 2.10
Deep Dive: Proof Expertise, Part One

Geometry GT

Task #1

Prove that opposite angles in an equilateral quadrilateral are congruent.

Task #2

Prove that if one diagonal of a quadrilateral is the perpendicular bisector of the other diagonal, then two pairs of adjacent sides are congruent.

Name: ______________________

Lesson 2.11
Deep Dive: Proof Expertise, Part Two

Geometry GT

Task #1

Prove that if a quadrilateral has two pairs of opposite sides that are congruent, then it is a parallelogram.

Task #2

Prove that if the diagonals of a quadrilateral are both perpendicular bisectors of each other, then the quadrilateral is a rhombus.