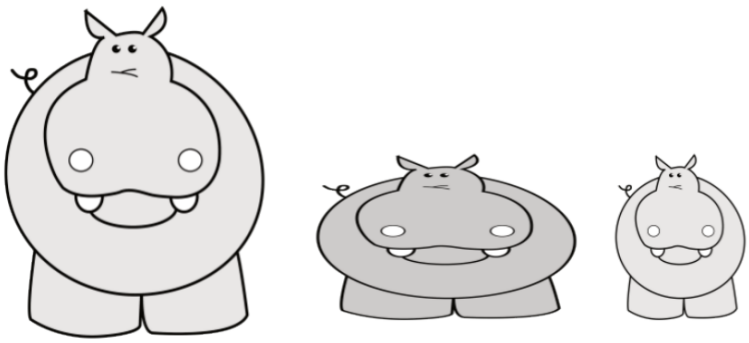
Name: _____

Lesson 3.01
Scaling Figures

Geometry GT

Analyze

Joaquin took a picture of a hippo and then edited it.



Which is the distorted image? How can you tell? Is there anything about the pictures you could measure to test whether there's been a distortion?

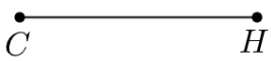
Definitions

Scale factor: the factor by which every length in an original figure is multiplied when you make a scaled copy

Dilation: a transformation that takes a point A along the ray $\overrightarrow{PA}$, where point P is the center of the dilation, to another point whose distance is k times farther away from P than A is

Explore

Measure the length of $\overline{CH}$, then dilate H using C as the center and a scale factor of 3.

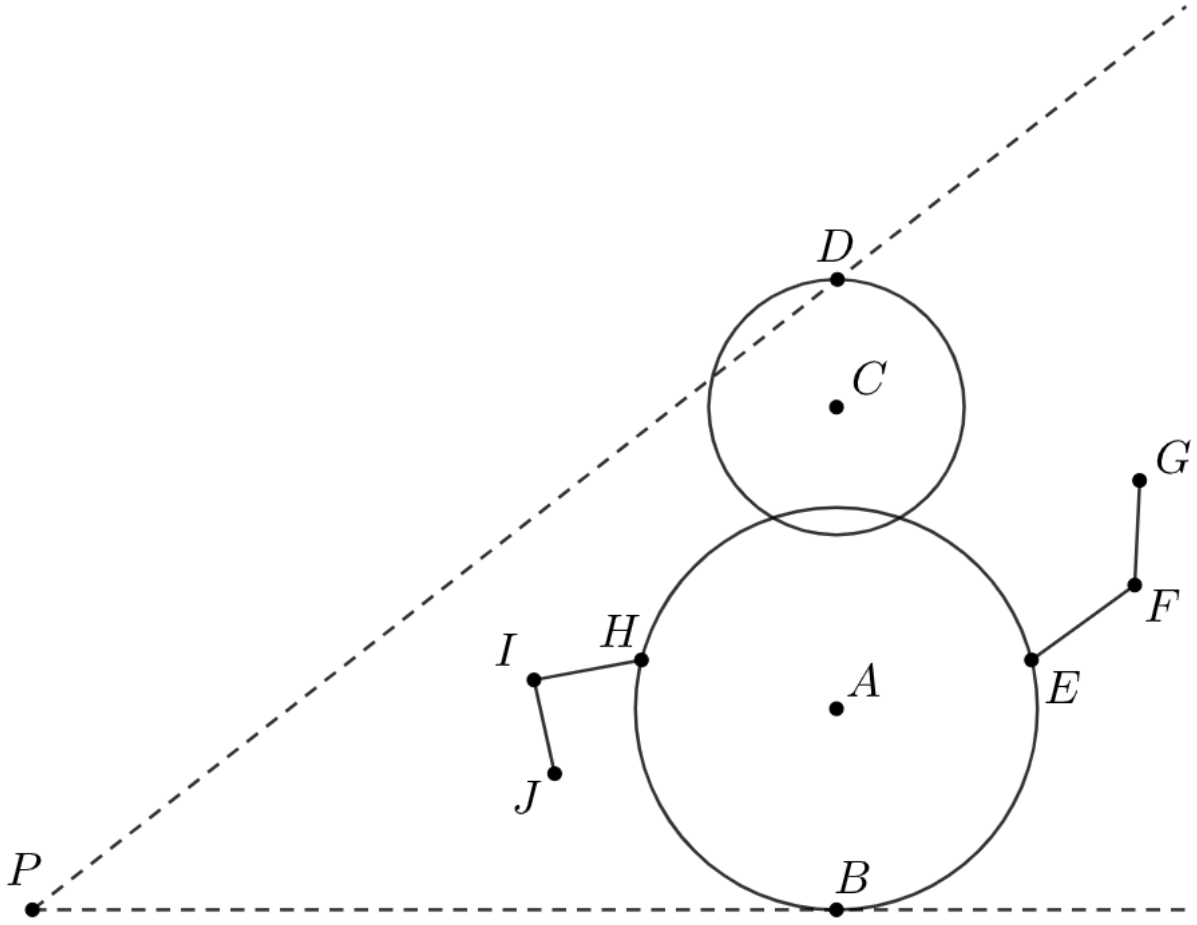


Measure the length of $\overline{KO}$, then dilate K using O as the center and a scale factor of $\frac{3}{4}$.

What difference do you notice between the two dilations?

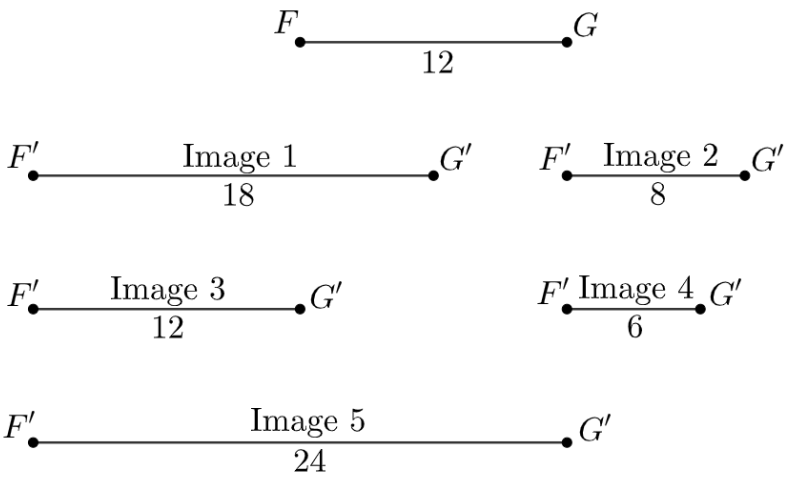
Discuss

Dilate the figure using center P and scale factor $\frac{1}{3}$.



Demonstrate

Match the image to the scale factor from $\overline{FG}$ to $\overline{F'G'}$.



A. 1

B. $\frac{3}{2}$

C. 2

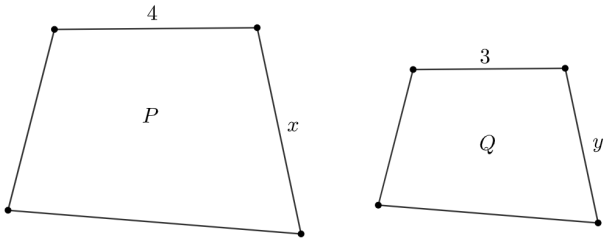
D. $\frac{2}{3}$

E. $\frac{1}{2}$

Practice

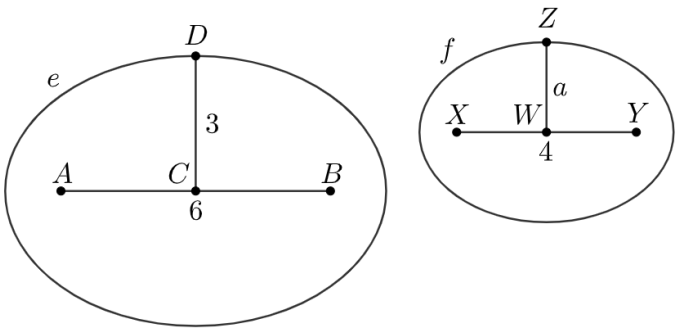
1. Polygon Q is a scaled copy of polygon P.

A. If the value of x is 6, what is the value of y?



B. What is the scale factor?

2. Figure f is a scaled copy of figure e.



We know:

- $AB = 6$
- $CD = 3$
- $XY = 4$
- $ZW = a$

Select **all** true equations.

A. $\frac{6}{3} = \frac{4}{a}$

B. $\frac{6}{4} = \frac{3}{a}$

C. $\frac{3}{4} = \frac{6}{a}$

D. $\frac{6}{3} = \frac{a}{4}$

E. $\frac{6}{4} = \frac{a}{3}$

F. $\frac{3}{4} = \frac{a}{6}$

3. Solve each equation.

A. $\frac{2}{5} = \frac{x}{15}$

B. $\frac{4}{3} = \frac{x}{7}$

C. $\frac{7}{5} = \frac{28}{x}$

D. $\frac{11}{4} = \frac{5}{x}$

Name: _____

Lesson 3.02
Deep Dive: Scaling the Solar System

Geometry GT

Task #1

Today we will make scale drawings of the planets in the solar system and their distances from Earth. To begin, a circle with a diameter of 2 cm represents Earth.

You will be assigned three planets. Complete the corresponding rows in the table below.

Planet	Average Diameter (km)	Average Orbit Radius (km)	Scaled Diameter (cm)	Scaled Orbit Radius (cm)	Scaled Distance from Earth (cm)
Mercury	4,879	57,900,00			
Venus	12,104	108,200,000			
Earth	12,756	149,600,000	2.000		
Mars	6,792	227,900,000			
Jupiter	142,984	778,600,000			
Saturn	120,536	1,433,500,000			
Uranus	51,118	2,872,500,000			
Neptune	49,528	4,495,100,000			

Do you believe it is possible to complete a scaled drawing in class? Explain your reasoning.

Task #2

Let's try this again, but imagine that the Earth is about the size of the period at the end of this sentence, which is about 0.3 mm in diameter.

You will be assigned three planets. Complete the corresponding rows in the table below.

Planet	Average Diameter (km)	Average Orbit Radius (km)	Scaled Diameter (mm)	Scaled Orbit Radius (mm)	Scaled Distance from Earth (mm)
Mercury	4,879	57,900,00			
Venus	12,104	108,200,000			
Earth	12,756	149,600,000	0.3000		
Mars	6,792	227,900,000			
Jupiter	142,984	778,600,000			
Saturn	120,536	1,433,500,000			
Uranus	51,118	2,872,500,000			
Neptune	49,528	4,495,100,000			

Can the new scale model fit inside the classroom?

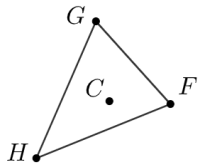
Name: _____

Lesson 3.03
Dilations

Geometry GT

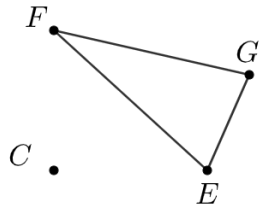
Recall

Dilate triangle ΔFGH using center C and a scale factor of 3.



Explore

Here is a center of dilation and a triangle.



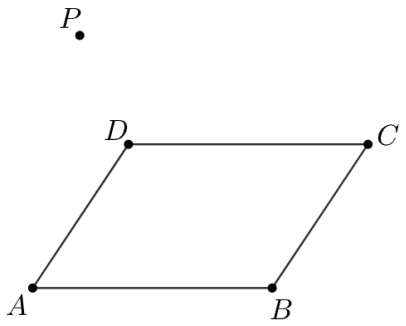
A. Measure the sides of ΔEFG (to the nearest mm).

B. Dilate ΔEFG using center C and your scale factor.

C. Compare your image with those around you. What do you notice?

Discuss

Dilate quadrilateral $ABCD$ using center P and your scale factor from the previous exercise.



Complete the table.

Ratio	$\frac{PA'}{PA}$	$\frac{PB'}{PB}$	$\frac{PC'}{PC}$	$\frac{PD'}{PD}$
Value				

What do you notice?

Complete the table.

Ratio	$\frac{B'A'}{BA}$	$\frac{C'B'}{CB}$	$\frac{D'C'}{DC}$	$\frac{A'D'}{AD}$
Value				

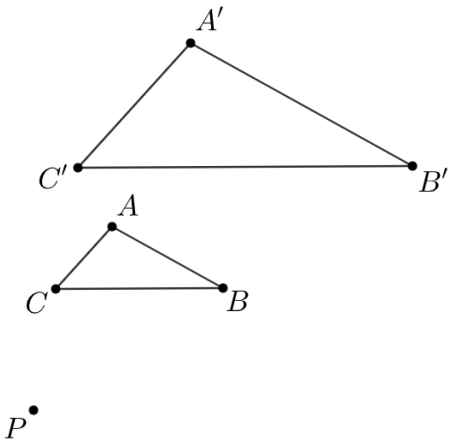
What do you notice?

Assertion

If a figure is dilated, then corresponding angles are congruent.

Demonstrate

Jada dilated ΔABC using center P and scale factor 2.



Jada claims that all the segments in ΔABC are parallel to the corresponding segments in $\Delta A'B'C'$.

Prove that this is true.

Theorem

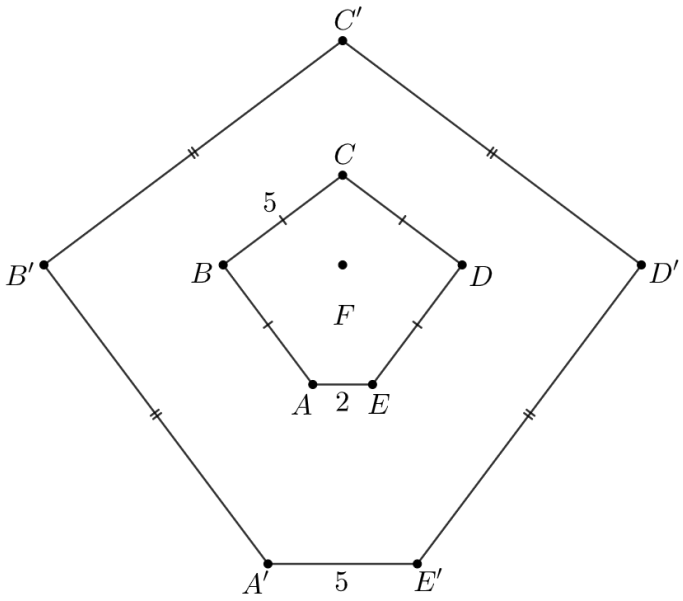
A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

Practice

1. Pentagon $A'B'C'D'E'$ is the image of pentagon $ABCDE$ after a dilation centered at F.

A. What is the scale factor of this dilation?

B. What is the length of $\overline{D'E'}$?



2. Triangle ΔABC is taken to triangle $A'B'C'$ by a dilation. Which of these scale factors for the dilation would result in an image that was *larger* than the original figure?

A. $\frac{3}{5}$

B. $\frac{13}{17}$

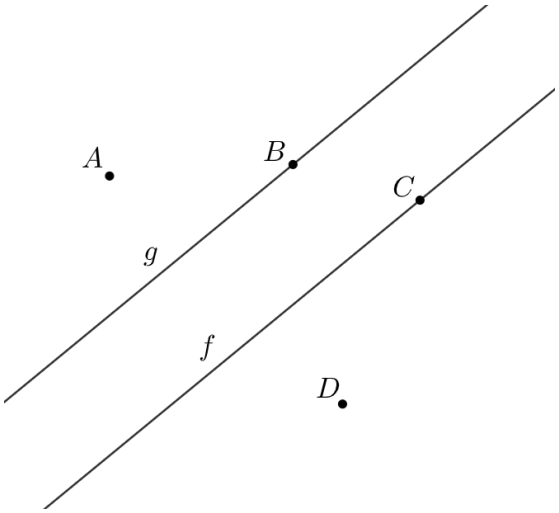
C. 1

D. $\frac{4}{3}$

3. A polygon has perimeter 12 units. It is dilated with a scale factor of $\frac{3}{4}$. What is the perimeter of its image?

4. Triangle ΔABC is taken by a dilation with center P and scale factor 3 to triangle $\Delta A'B'C'$. The measure of angle $\angle ABC$ is 21°. What is the measure of $\angle A'B'C'$?

5. Line f is dilated with a scale factor of 2, and the image is line g. Which labeled point could be the center of this dilation?

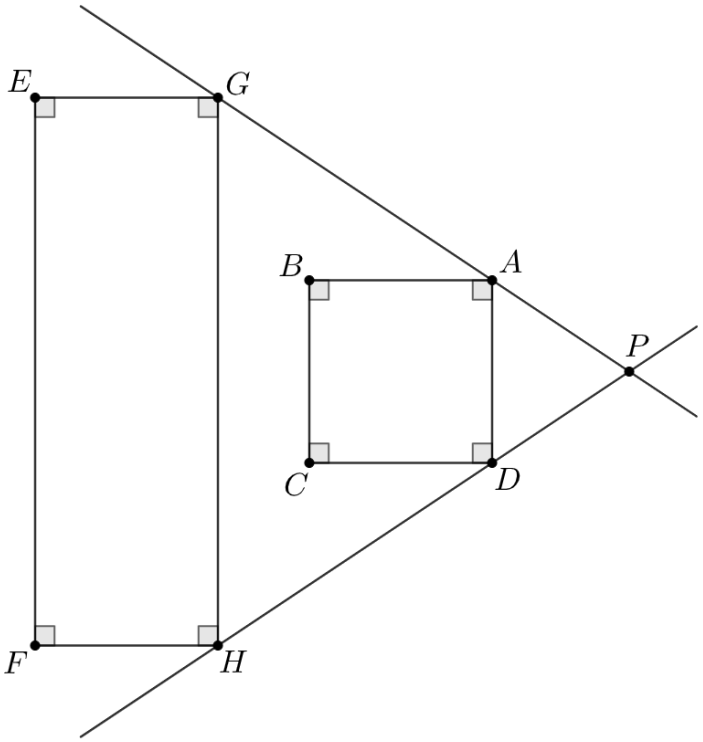


Name: _____

Lesson 3.04
Similar Figures

Geometry GT

Recall



What's wrong with this dilation? Why is $GHFE$ not a dilation of $ADCB$?

Definition

Similar: two figures with the same shape and proportional sides; if there exists a sequence of rigid motions and dilations that takes one figure onto another, then the two figures are similar

Explore

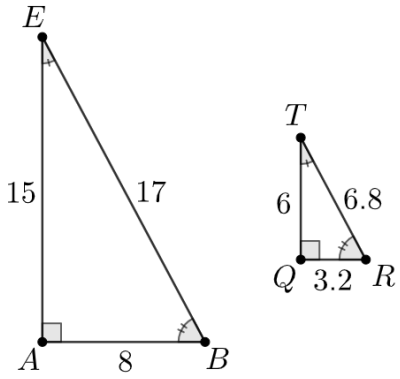
Sketch two triangles with all pairs of corresponding angles congruent, and with all pairs of corresponding side lengths in the same proportion. Label the triangles ΔABC and ΔDEF such that $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$. Label each side with its length (to the nearest millimeter).

Do the two triangles you drew fit the definition of similar? If so, find a sequence of rigid motions and dilations that will take ΔABC to ΔDEF.

Swap triangles with a neighbor. Does your sequence work for their figures?

Discuss

The following triangles are similar.



Write a similarity statement about the 2 figures, and explain how you know they are similar.

Compare your statement with a neighbor's statement. Is there more than one correct way to write a similarity statement? Is there a wrong way to write a similarity statement?

Demonstrate

Determine if each statement must be true, could possibly be true, or definitely can't be true. Explain or show your reasoning.

A. Congruent figures are similar.

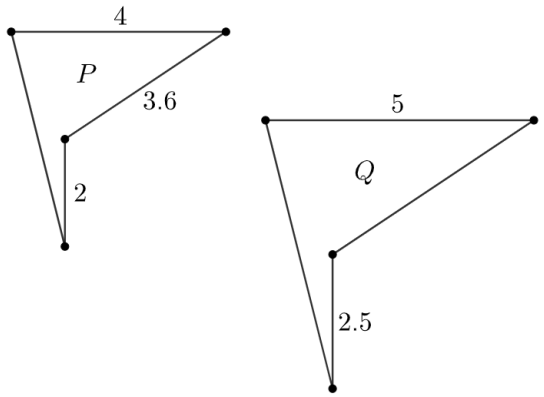
B. Similar figures are congruent.

Practice

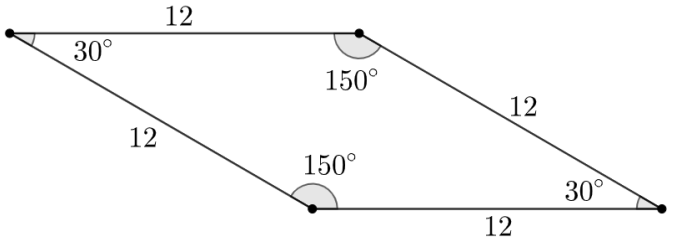
1. Quadrilaterals Q and P are similar.

A. What is the scale factor of the dilation that takes P to Q?

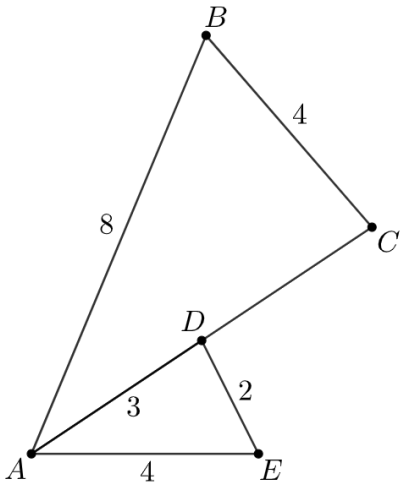
B. What is the scale factor of the dilation that takes Q to P?



2. Sketch a figure that is similar to the given figure. Label side and angle measures.



3. Write two different sequences of transformations that would show that ΔABC and ΔAED are similar. The length of $\overline{AC}$ is 6 units.



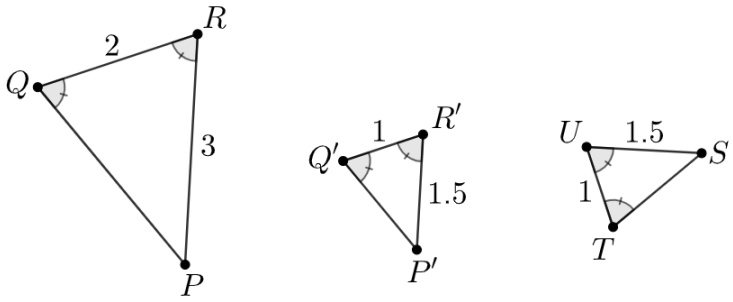
Name: _____

Lesson 3.05
AA Similarity

Geometry GT

Recall

How could you justify each statement?



Triangle $\Delta P'Q'R'$ is congruent to triangle ΔSTU.

Triangle ΔPQR is similar to triangle ΔSTU.

Explore

For parts **A** through **C**, draw two triangles that have the listed properties. Try to make them as different as possible.

A. One angle is 45°.

B. One angle is 45° and another angle is 30°.

C. One angle is 45° and another angle is 30°. The lengths of a pair of corresponding sides are 2 cm and 6 cm.

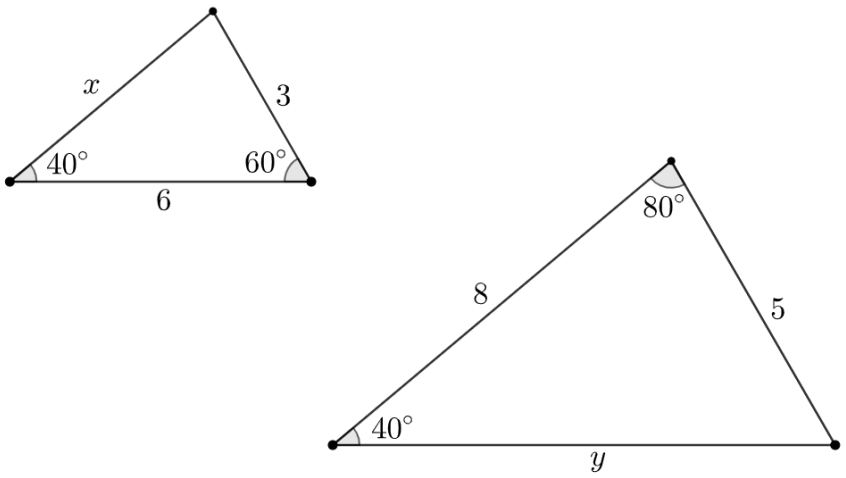
Compare your triangles with your neighbors' triangles. Which ones seem to be similar no matter what? Can you prove it?

Theorem

Angle-Angle Triangle Similarity Theorem: in two triangles, if two pairs of corresponding angles are congruent, then the triangles must be similar

Discuss

One triangle has a 60° angle and a 40° angle. Another triangle has a 40° angle and an 80° angle.



Explain how you know the triangles are similar.

How long are the sides labeled x and y?

Demonstrate

Vivian noticed in the previous activity that between the two triangles, you only need to know 4 angles to show that they are similar. She wondered which fourth angle would work to prove $\Delta RST \sim \Delta EFG$.

In ΔRST: $m\angle R = 90°$, $m\angle S = 25°$, $m\angle T = x°$

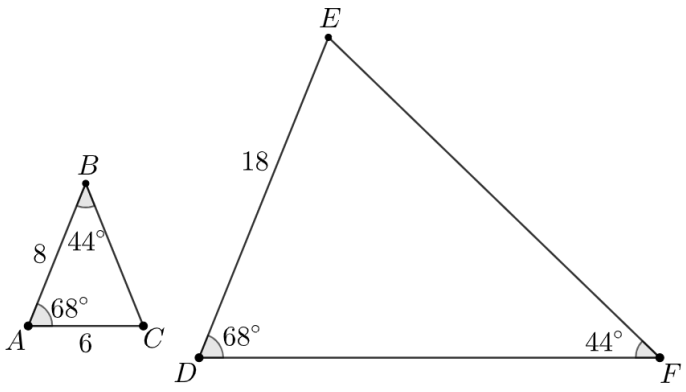
In ΔEFT: $m\angle E = 90°$, $m\angle F = y°$, $m\angle G = z°$

Draw a sketch of the triangles, then pick one angle measurement that would prove the triangles are similar.

Explain to Vivian why knowing that angle would be enough.

Practice

1. What is the length of $\overline{DF}$?



2. In ΔABC, $m\angle A = 35°$ and $m\angle B = 20°$. Select **all** triangles which are similar to ΔABC.

A. ΔDEF with $m\angle D = 35°$ and $m\angle E = 20°$

B. ΔGHI with $m\angle G = 35°$ and $m\angle I = 30°$

C. ΔJKL with $m\angle J = 35°$ and $m\angle L = 125°$

D. ΔMNO with $m\angle N = 20°$ and $m\angle O = 125°$

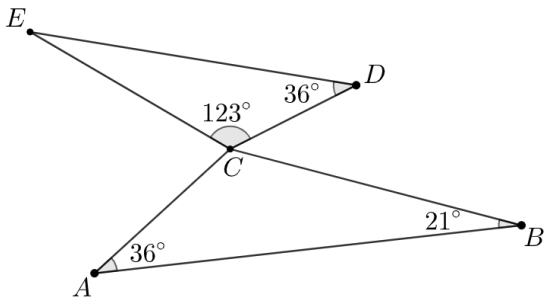
E. ΔPQR with $m\angle Q = 20°$ and $m\angle R = 30°$

3. Determine if each statement must be true, could possible be true, or definitely can't be true.

A. An equilateral triangle and a right triangle are similar

B. A right triangle and an isosceles triangle are similar

4. Determine if $\Delta ABC \sim \Delta DEC$. Explain or show your reasoning.



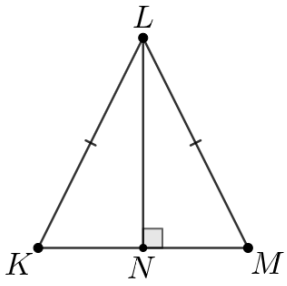
Name: _____

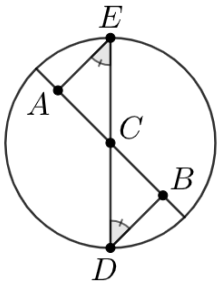
Lesson 3.06
SAS/SSS Similarity

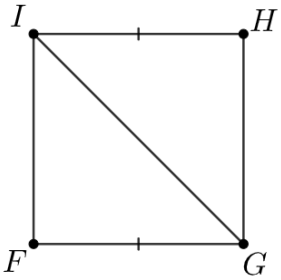
Geometry GT

Recall

Is there enough information to determine if the pairs of triangles are congruent? If so, what theorem(s) would you use? If not, what additional piece of information could you use?







Explore

Recall that there are five ways to prove triangles congruent.

Could you use Angle-Side-Angle and Angle-Angle-Side to prove triangles congruent? Why or why not?

What about Side-Angle-Side and Side-Side-Side? What could "side" mean in the context of similar triangles?

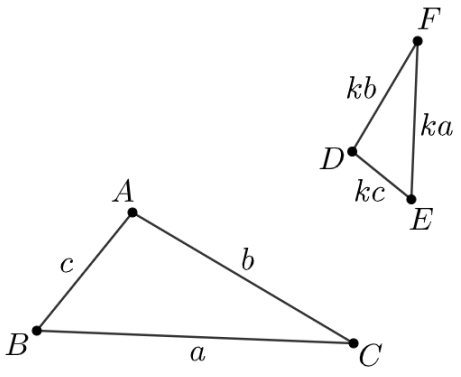
Sketch a diagram to illustrate Side-Angle-Side and find a sequence of rigid motions and dilations to take one to the other.

Theorem

Side-Angle-Side Triangle Similarity Theorem: in two triangles, if two pairs of corresponding sides are in the same proportion, and the corresponding pair of included angles are congruent, then the two triangles are similar

Discuss

Explain why these two triangles must be similar.

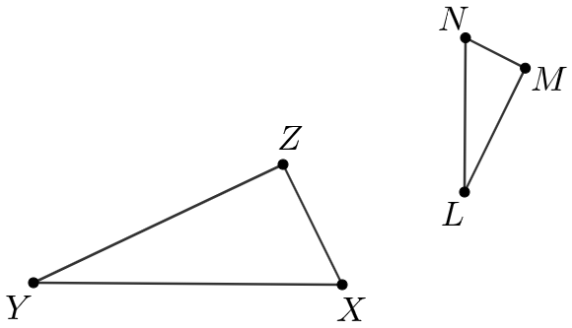


Theorem

Side-Side-Side Triangle Similarity Theorem: in two triangles, if all three pairs of corresponding sides are in the same proportion, then the two triangles are similar

Demonstrate

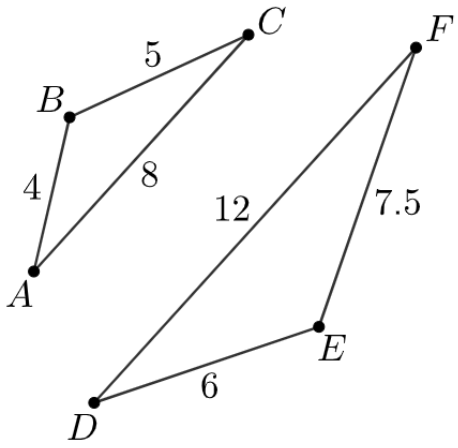
Invent measurements that would show $\Delta XYZ \sim \Delta NLM$ using the Side-Angle-Side Triangle Similarity Theorem.



Practice

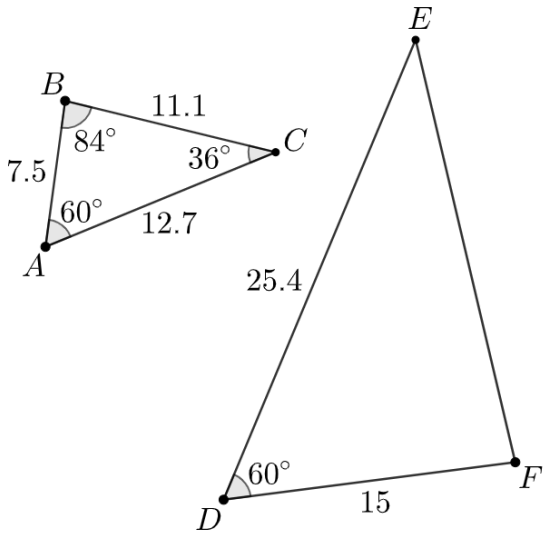
1. Here are triangles ΔABC and ΔDEF.

A. Explain how we know $\Delta ABC \sim \Delta DEF$.



B. What does that tell us about $\angle D$?

2. Here are two similar triangles, ΔABC and ΔDFE.

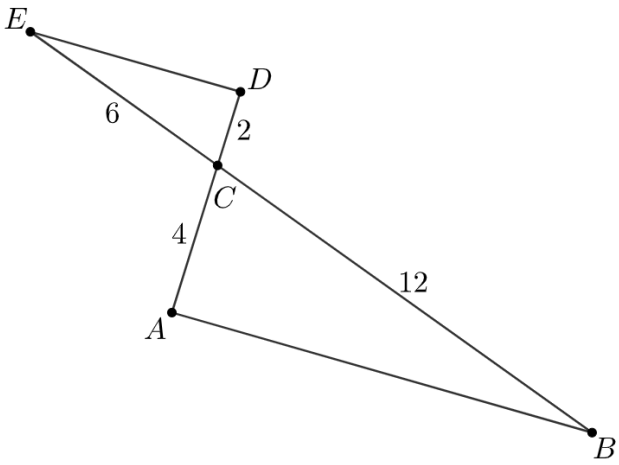


A. Find the length of $\overline{EF}$.

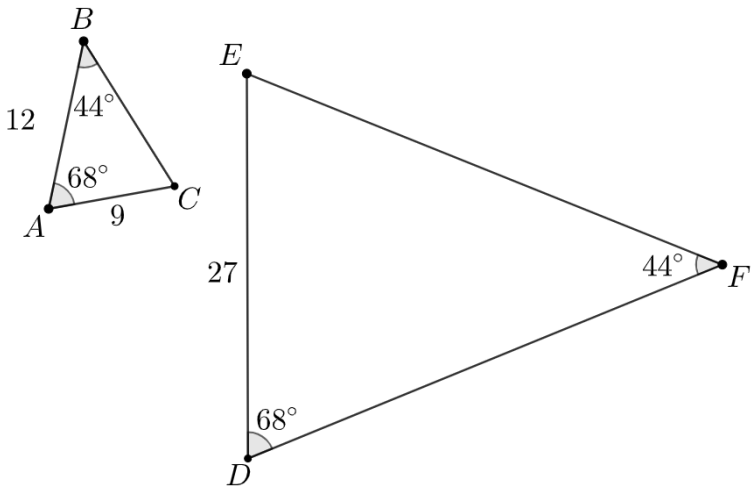
B. Find the measure of $\angle E$.

C. Find the measure of $\angle F$.

3. Prove $\Delta ABC \sim \Delta DEC$.



4. What is the length of $\overline{DF}$?



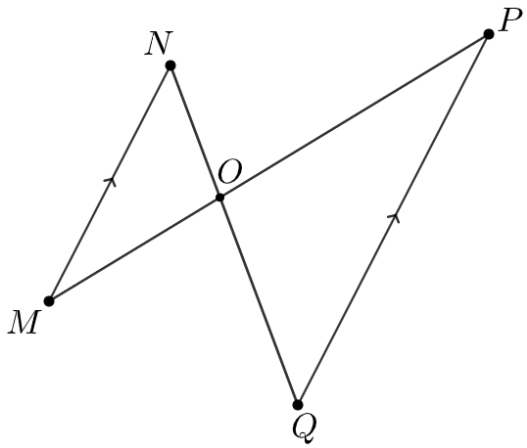
Name: _____

Lesson 3.07
Deep Dive: Similar Triangle Proofs

Geometry GT

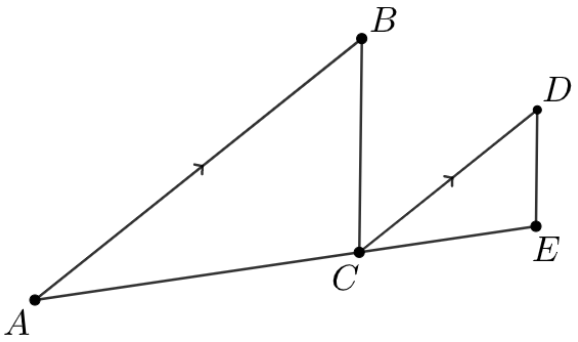
Task #1

Given that $\overline{MN} \parallel \overline{PQ}$, prove that $\Delta MNO \sim \Delta PQO$.



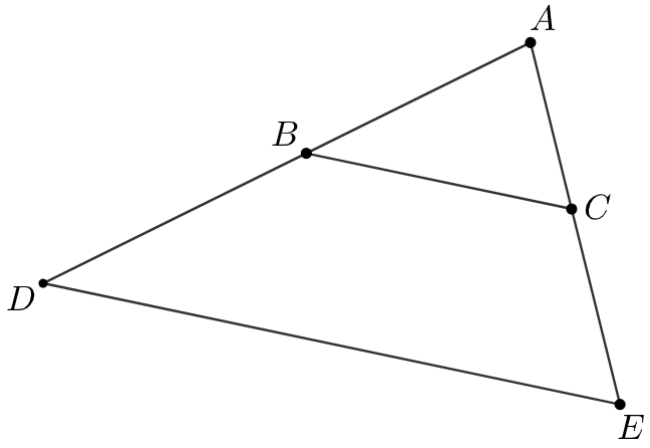
Task #2

Given that $\frac{AB}{CD} = \frac{AC}{CE}$ and $\overline{AB} \parallel \overline{CD}$, prove that $\Delta ABC \sim \Delta CDE$.



Task #3

Given that $\frac{AB}{AD} = \frac{AC}{AE}$, prove that $\overline{BC} \parallel \overline{DE}$.



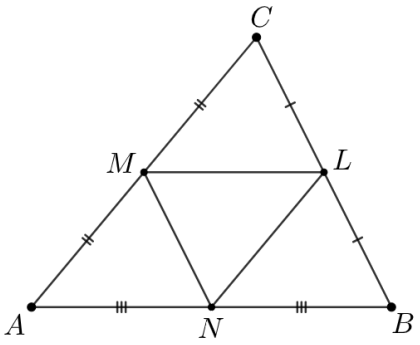
Name: _____

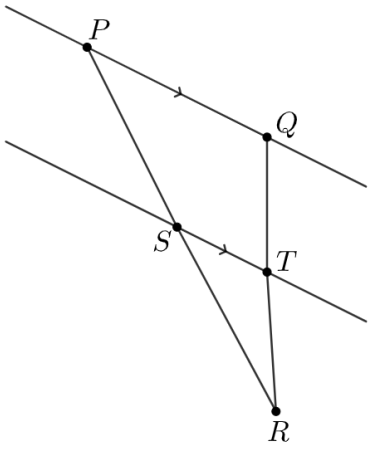
Lesson 3.08
Midsegments & Parallel Lines

Geometry GT

Analyze

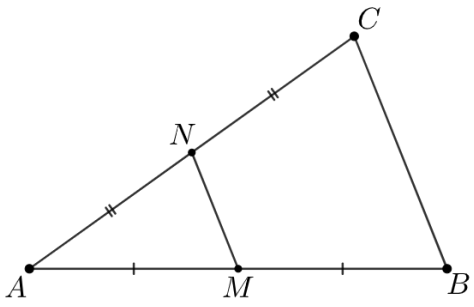
For each figure, answer the following questions: What do you notice? What do you wonder?





Explore

Here is triangle ΔABC with point M as the midpoint of $\overline{AB}$ and point N as the midpoint of $\overline{AC}$.



ΔABC is a dilation of ΔAMN. What is the center of the dilation? What is the scale factor?

Explain why $BC = 2MN$ must be true.

Explain why $\overline{MN}$ must be parallel to $\overline{BC}$.

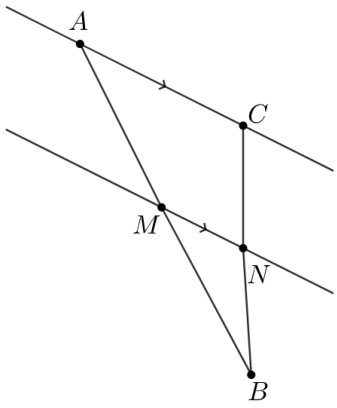
Theorem

If a line divides two sides of a triangle proportionally, then the line must be parallel to the third side of the triangle

Discuss

Does a line parallel to one side of a triangle always create similar triangles? Create several examples, and consider if there are any additional requirements to create similar triangles.

Using the diagram below, find any additional information you can be sure is true, and explain why similar triangles are always created by this parallel line.

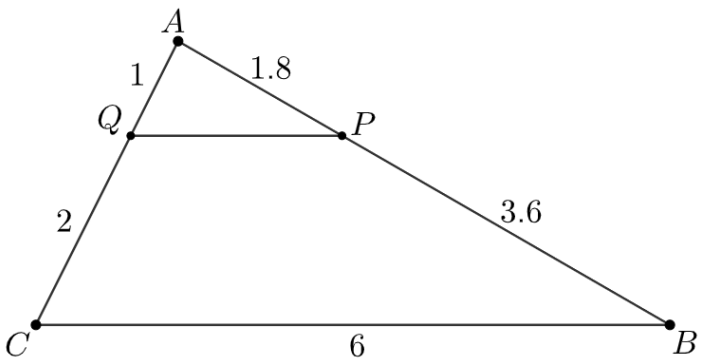


Theorem

If a line intersects two sides of a triangle and is parallel to the third side, then the line must create a triangle similar to the original triangle

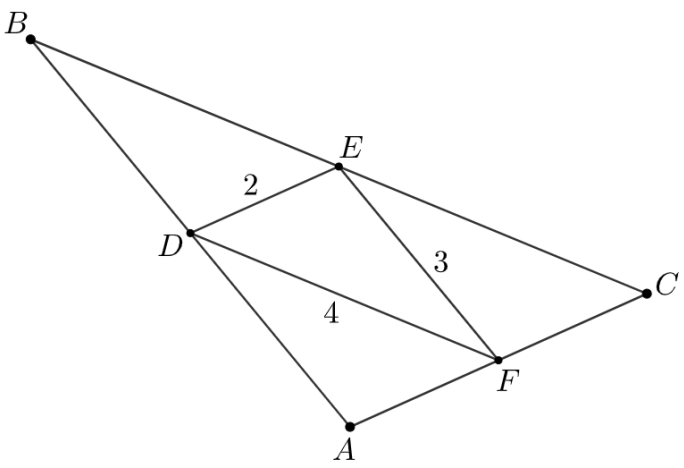
Demonstrate

What must be true about $\overline{PQ}$?



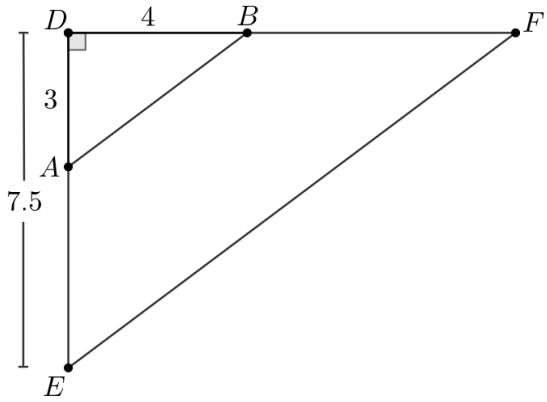
Practice

1. ΔDEF is formed by connecting the midpoints of the sides of ΔABC. The lengths of ΔDEF are shown. What is the length of $\overline{AB}$?

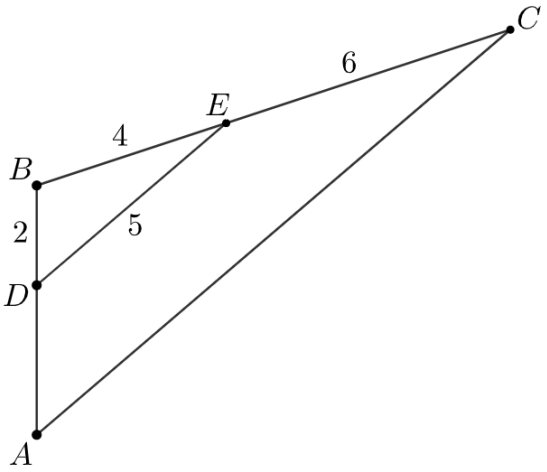


For problems #2-3, find the length of each unlabeled side.

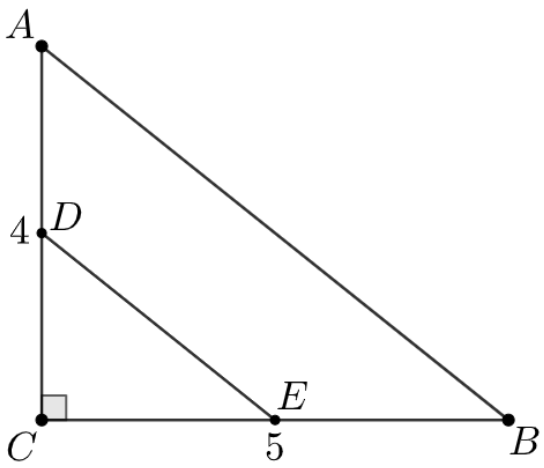
2. Segments $\overline{AB}$ and $\overline{EF}$ are parallel.



3. Segments $\overline{DE}$ and $\overline{AC}$ are parallel.



4. In right triangle ΔABC, $AC = 4$ and $BC = 5$. A new triangle ΔDEC is formed by connecting the midpoints of $\overline{AC}$ and $\overline{BC}$.



A. What is the area of ΔABC?

B. What is the area of ΔDEC?

C. Does the scale factor for the side lengths apply to the area as well? Explain.

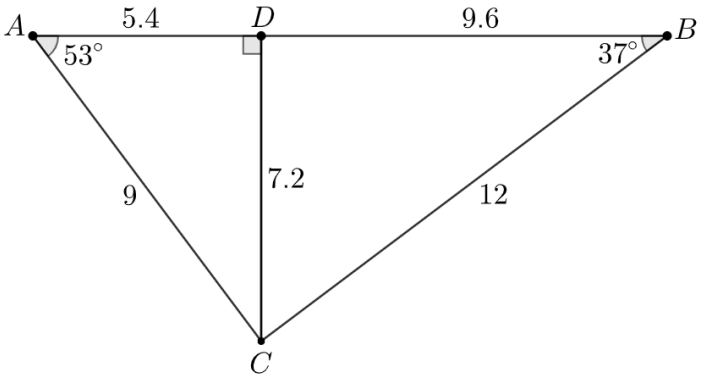
Name: _____

Lesson 3.09
Pythagorean Theorem

Geometry GT

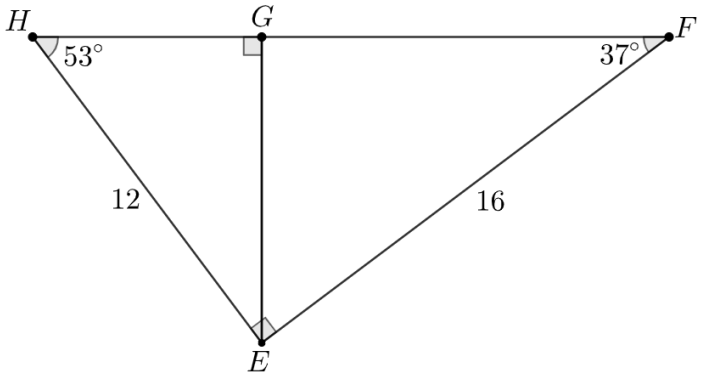
Analyze

Is $\Delta ADC \sim \Delta CDB$? Explain or show your reasoning.



Explore

Trace the two smaller triangles onto separate sheets of patty paper.

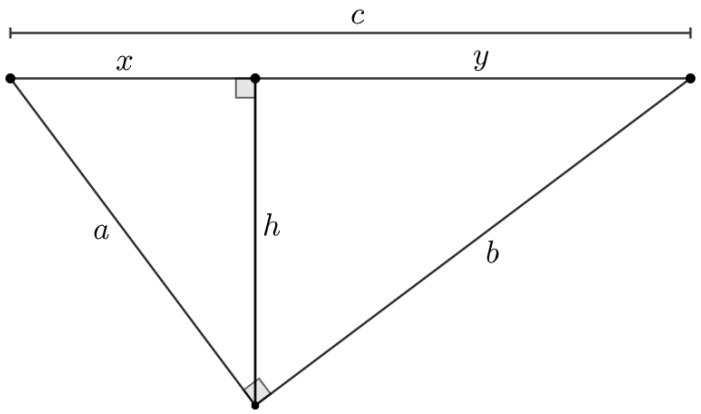


Rotate your patty paper and convince yourself that all three triangles are similar, then write three similarity statements.

Determine the scale factor for each pair of triangles.

Determine the lengths of sides $\overline{HG}$, $\overline{GF}$, $\overline{HF}$.

Discuss



Since all three triangles are similar, the following equivalent ratios can be formed: $\frac{a}{x} = \frac{c}{a}$ and $\frac{b}{y} = \frac{c}{b}$.

Use cross multiplication to simplify each equivalence.

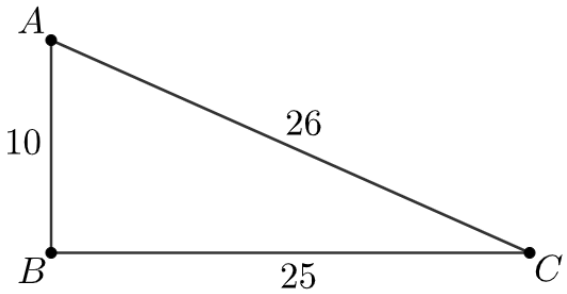
Prove that $a^2 + b^2 = c^2$ using your answers from above.

Theorem

Pythagorean Theorem: if a right triangle has legs with length a and b and hypotenuse with length c, then $a^2 + b^2 = c^2$

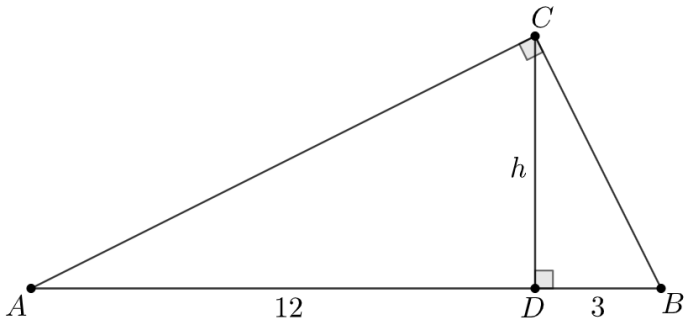
Demonstrate

Is ΔABC a right triangle? Explain.

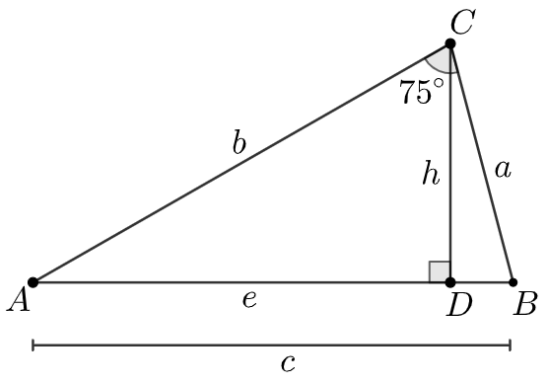


Practice

1. In right triangle ΔABC, altitude $\overline{CD}$ with length h is drawn to its hypotenuse. We also know $AD = 12$ and $DB = 3$. What is the value of h?



2. In ΔABC (*not* a right triangle), altitude $\overline{CD}$ is drawn to side $\overline{AB}$. The length of $\overline{AB}$ is c. Which of the following statements must be true?



A. $m\angle ACB = m\angle ABC$

B. $b^2 = c^2 + a^2$

C. $\Delta ADC \sim \Delta ACB$

D. The area of ΔABC is $\frac{1}{2}h \cdot c$

3. Which of the following are right triangles?

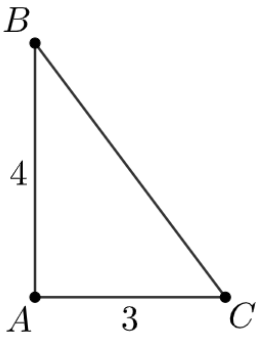
A. ΔABC with $AC = 6$, $BC = 9$, and $AB = 12$

B. ΔDEF with $DE = 8$, $EF = 10$, and $FD = 13$

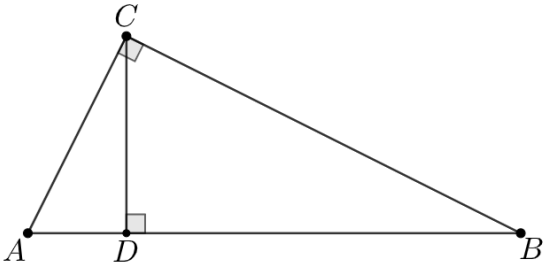
C. ΔGHI with $GI = 9$, $HI = 12$, and $GH = 15$

D. ΔJKL with $JL = 10$, $KL = 13$, and $JL = 17$

4. Moura says she can find the length of the third side of ΔABC and it is 5 units. Rowan disagrees and thinks that the side length is unknown. Do you agree with either of them? Show or explain your reasoning.



5. In right triangle ΔABC, altitude $\overline{CD}$ is drawn to its hypotenuse. Find two triangles which must be similar to ΔABC.



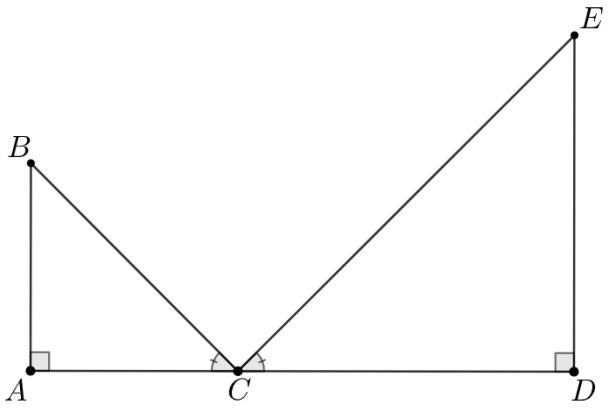
Name: _____

Lesson 3.10
Deep Dive: Practice with Similarity

Geometry GT

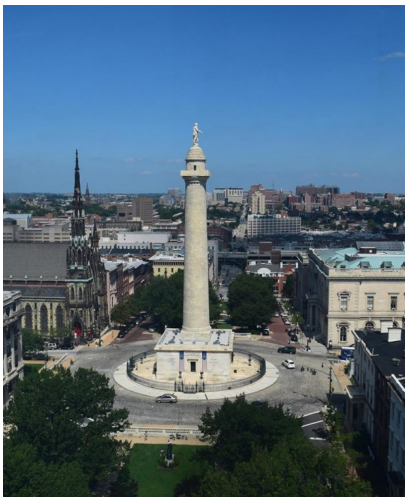
Analyze

What do you notice? What do you wonder?



Task

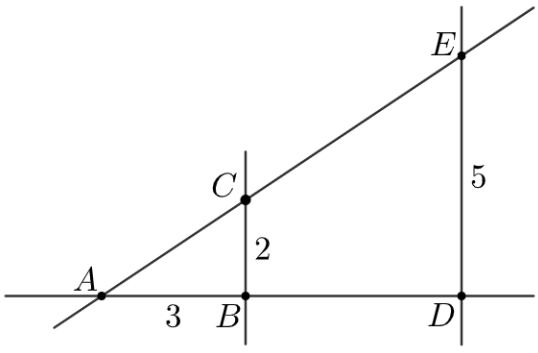
The first major monument to honor George Washington is the Washington Monument, located north of Baltimore and built in 1829.



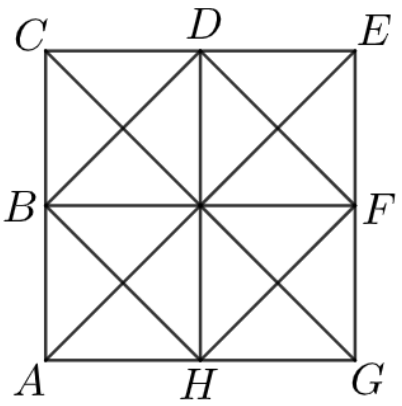
Andrea is 5.5 feet tall and sets a mirror on the ground 3 feet away from her, such that she can see the top of the Washington Monument. She measures the distance from the mirror to the monument as 97.5 feet. How tall is the Washington Monument?

Practice

1. Lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{DE}$ are vertical. What is the length of $\overline{BD}$?



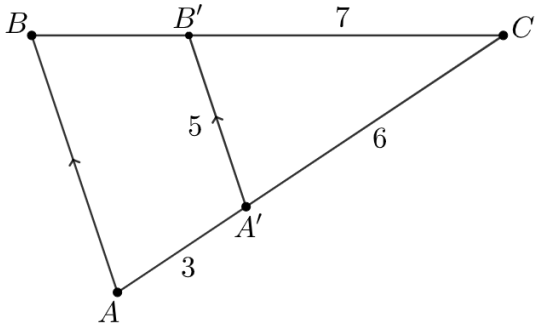
2. The quilt is made of squares with diagonals. Side length $\overline{AB}$ has a length of 2.



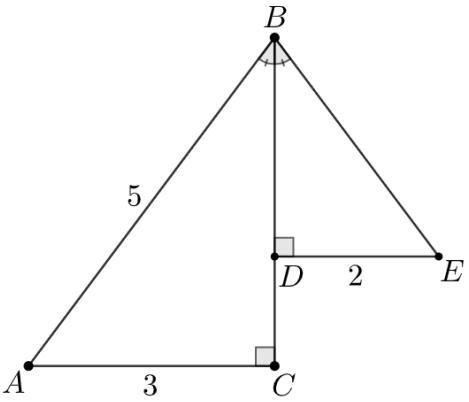
A. What is the length of $\overline{BD}$?

B. What is the area of ΔAEH?

3. Segment $\overline{A'B'}$ is parallel to segment $\overline{AB}$. What is the length of segment $\overline{BB'}$?

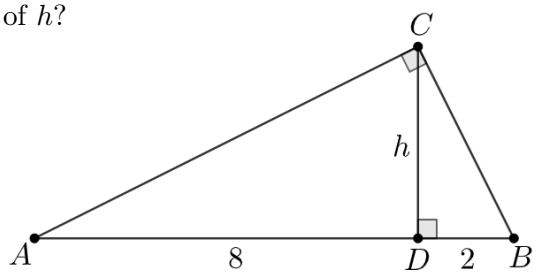


4. In the right triangles shown, $\angle ABC \cong \angle EBD$. What is the length of BE?



5. In right triangle ΔABC, $\angle C$ is a right angle, $AB = 13$, and $BC = 5$. What is the length of $\overline{AC}$?

6. In right triangle ΔABC, altitude $\overline{CD}$ with length h is drawn to its hypotenuse. Given $AD = 8$ and $DB = 2$, what is the value of h?



7. Harper is playing a game with a class of second graders. Harper knows they are exactly 120 inches from the mirror on the floor. Students stand so that Harper can just see the top of their heads and the guesses their heights. The students are amazed!

A. How is Harper so accurate? Draw a diagram to support your reasoning.

B. If a student is 72 inches away from the mirror, and Harper is 65 inches tall, how tall is the student?